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A GENERAL FAILURE CRITERION FOR GEOMATERIALS WITH CROSS ANISOTROPY

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ABSTRACT: *The strength of soils and rocks is greatly influenced by cross anisotropy which cannot be properly characterized by an isotropic failure criterion. A novel anisotropic failure criterion is proposed for geomaterials in this paper. To account for the influence of cross anisotropy, an anisotropic variable defined by the invariants and joint invariants of the stress tensor and the fabric tensor is introduced into the expression of the criterion. We formulate the new anisotropic failure criterion in both the deviatoric plane and the meridian plane, which collectively lends a general three-dimensional description for the strength anisotropy. All parameters in the criterion can be conveniently determined by conventional laboratory tests. By comparing the model predictions with experimental data, we demonstrate that the new criterion is general and robust enough to characterize the strength anisotropy for a wide range of materials, including clays, sands and rocks. Further extension of the failure criterion for constitutive modeling of geomaterials is discussed.*

1 INTRODUCTION

Cross anisotropy is commonly observed in geomaterials and has been known to have a significant influence on the strength of these geomaterials (Casagrande & Carrilo, 1944; Duncan & Seed, 1966; Arthur & Menzies, 1972; Oda et al., 1978; Abelev & Lade, 2004). Numerous experimental data have proved that the strength of geomaterials varies considerably with direction. Yong & Silvest (1979), for example, have found the unconfined compression strength of sensitive clay varies continuously with loading directions and the minimum strength observed is about 60% to 75% of the maximum. Similar observations were reported by Kirkgard & Lade (1991, 1993) from true triaxial tests on San Francisco Bay Mud, by Nishimura et al. (2007) on natural London clay and by Niandou et al. (1997) and Duveau et al. (1998) on sedimentary rocks. The degree of strength anisotropy (e.g., difference in strength at different direction) is significantly higher in rocks than in soils. Indeed, strength anisotropy of geomaterials is important to a variety of geotechnical structures, such as footings, retaining walls and slopes. Oda et al. (1978) have investigated the bearing capacity of two model strip foundations built on the same sand, and found the difference in bearing capacity for the model with load perpendicular to the bedding plane and the other one with a parallel load to the bedding plane can reach as much as 34%. Proper consideration of cross anisotropy in the evaluation of the soil/rock strength appears to be necessary.

Most well-received failure criteria in the literature have been isotropic ones (e.g., Matsuoka & Nakai, 1974; Lade & Duncan, 1975; Lade, 1977; van Eekelen, 1980). Despite their popularity, these isotropic criteria may find difficulties in interpreting the yielding and failure for anisotropic soils/rocks. In a comparison of experimental data on isotropically consolidated San Francisco Bay Mud against predictions by Lade's (1977) isotropic failure criterion, Kirgard & Lade (1993) have found Lade's isotropic criterion can fit reasonably well for the failure stress points of specimens with Lode's angle θ in the range from 0° to 90° in the octahedral plane, but leads to An appreciable discrepancy, however, has been observed between Lade's failure surface and the failure data points for tests conducted with θ greater than 90° wherein Lade's failure criterion generally overestimates the strength of the specimen. An anisotropic failure criterion is indeed necessary to take into account the influence of cross anisotropy in cases like this.

In this paper, we propose a novel anisotropic failure criterion. As will be shown in the following section, this new criterion differs in essence from the existing anisotropic failure criteria appearing recently, such as those proposed by Duveau et al. (1998), Pietruszczak & Mroz (2000, 2001), Liu & Carter (2003), Abeleve & Lade (2004), Guo & Stolle (2005), Lade (2007, 2008), Lee & Pietruszczak (2008), Mortara (2009) and Schweiger et al. (2009). All model parameters introduced in the new criterion can be conveniently calibrated by conventional laboratory tests. It is general and robust enough to be capable of describing the failure behavior for a wide range of soils and rocks with cross-anisotropy.

2 THE GENERAL ANISOTROPIC FAILURE CRITERION

We propose the following failure criterion for geomaterials with cross anisotropy

$$\alpha\sqrt{\bar{I}_1^2-3\bar{I}_2}+(1-\alpha)\frac{2\bar{I}_1}{3\sqrt{\left(\bar{I}_1\bar{I}_2-\bar{I}_3\right)/\left(\bar{I}_1\bar{I}_2-9\bar{I}_3\right)}-1}=M_f f(A)\,\bar{p} \tag{1}$$

where M_f is the frictional coefficient of the material which depends on its peak frictional angle φ. $\bar{I}_1$, $\bar{I}_2$ and $\bar{I}_3$ are the three invariants of a transformed stress tensor $\bar{\sigma}_{ij}$ defined below (see Fig.1a for the transformation based on the meridian plane)

$$\bar{\sigma}_{ij}=\sigma_{ij}+(\bar{p}-p)\delta_{ij}=\sigma_{ij}+\left[p_r\left(\frac{p+\sigma_0}{p_r}\right)^n-p\right]\delta_{ij} \tag{2}$$

where δ_{ij} is the Kronecker delta. σ_{ij} is the commonly referred Cauchy stress tensor. As is shown in Fig. 1(a) in the meridian plane, the transformation essentially renders that $\bar{q}=q$ and $\bar{p}=\bar{q}/M_f=p_r\left[\left(p+\sigma_0\right)/p_r\right]^n$, where p and q are the commonly referred mean stress and shear stress, and $\bar{p}$ and $\bar{q}$ are their corresponding transformed values. n is a exponential constant and p_r a reference pressure. σ_0 denotes the triaxial tensile strength of the material, or equivalently the cohesion of the material. α is an interpolation constant as shown in Fig. 1(b). If $\alpha=1$, the underlying isotropic failure criterion of Eq. (1) (e.g., when $f(A)=1$) becomes identical to the extended Mises criterion. When $\alpha=0$, it coincides with the Matsuoka-Nakai (or so-called Spatial Mobilized Plane, SMP in brief) curve-sided triangle (Matsuoka and Nakai, 1974). The anisotropic failure criterion proposed in Eq. (1) has been based on an isotropic one originally developed by Yao et al. (2004). Key to the new failure criterion is the addition of the function $f(A)$ defined below to introduced the influence of anisotropy

$$f(A) = \exp\left\{d\left[(A+1)^2 + \beta(A+1)\right]\right\} \tag{3}$$

where d and β are material constants. A is an anisotropic variable reflective of the influence of loading direction with respect to fabric, defined as follows

$$A = \frac{\mathrm{tr}\left(s_{ik}d_{kj}\right)}{\sqrt{s_{mn}s_{mn}}\sqrt{d_{pq}d_{pq}}} \tag{4}$$

where $s_{ij} = \sigma_{ij} - p\delta_{ij}$, $d_{ij} = F_{ij} - F_{kk}\delta_{ij}/3$. F_{ij} denotes a fabric tensor representing the inherent anisotropy in a material. For materials with cross anisotropy, it is common F_{ij} adopts the following form

$$F_{ij} = \begin{bmatrix} F_1 & 0 & 0 \\ 0 & F_2 & 0 \\ 0 & 0 & F_3 \end{bmatrix} = \frac{1}{3+\Delta}\begin{bmatrix} 1-\Delta & 0 & 0 \\ 0 & 1+\Delta & 0 \\ 0 & 0 & 1+\Delta \end{bmatrix} \tag{5}$$

where Δ is a scalar that characterizes the magnitude of the cross-anisotropy. Its value ranges from zero when the material is absolutely isotropic, to unity when the degree of anisotropy is the maximum. For materials with cross anisotropy, A in Eq. (4) can be rewritten specifically for different loading conditions, in terms of the intermediate principal stress ratio b and the relative angle between stress direction and axis of cross anisotropy (for detail, please refer to Gao et al., 2010). Note that when $f(A) \equiv 1$, the anisotropic failure criterion expressed in Eq. (1) becomes identical to the underlying isotropic failure criterion as in Yao et al.(2004). The primary effect of the function $f(A)$ is to change the shape of the underlying isotropic failure surface in the deviatoric plane. When $f(A) > 1$, it plays a role in Eq. (1) in expanding the failure surface with respect to the isotropic one, and to shrink it when $f(A) < 1$.

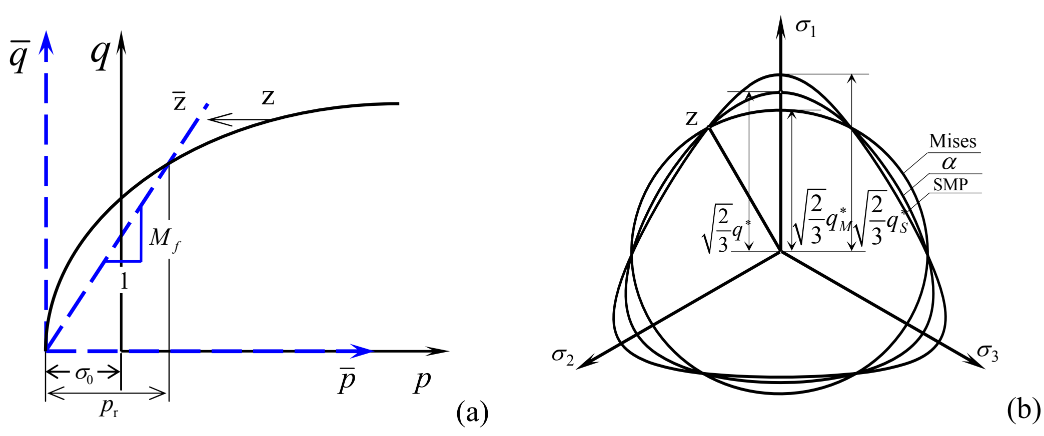


Fig. 1. Illustration of stress transformation in the meridian plane (a) and interpolation of failure surface in the deviatoric plane (b).

Model parameters introduced in the criterion can be calibrated using conventional laboratory tests, such as conventional and true triaxial compression/extension tests and rotational shear tests. Detailed procedures on their calibration can be referred to Gao et al. (2010), wherein specific examples on determining the model parameters for a soil or rock based on data from triaxial tests and/or rotational shear tests have been provided.

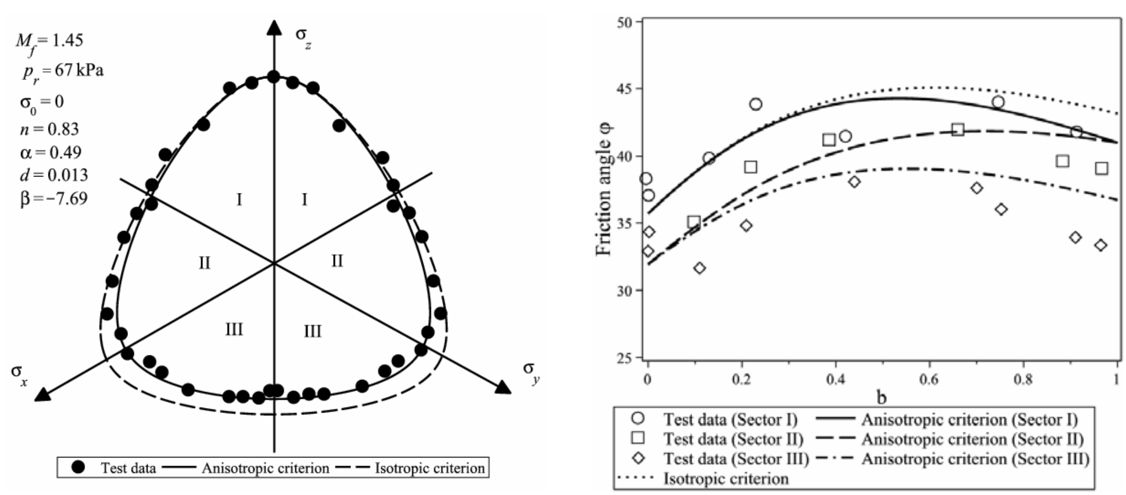
3 VALIDATION WITH EXPERIMENTAL DATA

To demonstrate the generality and robustness of our failure criterion, we have chosen a total of two clays, four sands and two rocks reported in the literature to validate our model prediction. According to the procedures outlined in Gao et al. (2010), the model parameters for these materials are calibrated and summarized in Tab. 1 below.

Table 1 Summary of model parameters calibrated for soils and rocks under study in this paper

Materials (Data source)		M_f	p_r	σ_0	n	α	d	β
Clay	Isotropically consolidated San Francisco Bay Mud (Kirkgard & Lade, 1993)	1.45	67 kPa	0	0.83	0.49	0.013	-7.69
	K_0-consolidated San Francisco Bay Mud (Lade & Kirkgard, 2000)	1.38	—*	0	1	0	0.058	1.44
Sand	Cambria Sand (Ochiai & Lade, 1983)	1.62	—	0	1	0.48	0.014	-3.57
	Dense Santa Monica Beach Sand (Abelev & Lade, 2004)	1.87	—	0	1	0.33	-0.05	-1
	Toyoura Sand (Lam and Tatsuoka, 1988; Tatsuoka et al., 1990)	1.68	—	0	1	0.17	0.05	-2.6
	Dry-pluviated Santa Monica Beach Sand (Lade et al., 2008)	1.63	—	0	1	0.36	-0.04	0
Rock	Touremire Shale (Niandou et al., 1997)	1.58	50 MPa	2.5 MPa	0.54	—	0.5	-1.5
	Angers Schist (Duveau et al., 1998)	2.36	100 MPa	8 MPa	0.76	—	2.5	-1.52

* — : not specified.



(a) (b)

Fig. 2. Comparison of the isotropic and anisotropic failure criteria with experimental data for isotropically consolidated San Francisco Bay Mud (Kirkgard & Lade, 1993), (a) in the deviatoric plane, and (b) in the φ-b diagram.

3.1 Isotropically Consolidated San Francisco Bay Mud (Kirkgard & Lade, 1993)

The anisotropic failure criterion proposed in Section 2 is first employed to predict the anisotropic strength of isotropically consolidated San Francisco Bay Mud tested by Kirkgard & Lade (1993). The test data are compared in Fig. 2 against the predictions by the anisotropic failure criteria in Eq. (1) as well as the underlying isotropic failure criterion (by using $f(A) \equiv 1$ and the same α value in Eq. (1)) in the deviatoric plane for which $p = 167$ kPa and the plane of $\varphi \sim b$ where φ is the peak friction angle of the soil. As is shown, the anisotropic criterion captures the overall trend of the test data in the deviatoric plane reasonably well, with only a slight underestimation of the soil strength in Sector II (see Fig. 2(a) for the different sections defined in the deviatoric plane. The same zonation of sections is followed for all subsequent figures on the description of deviatoric plane). In contrast, the isotropic criterion clearly overestimates the strength at large, particularly in Sector III. The tested $\varphi \sim b$ relation and the corresponding predictions of the two failure criteria are also shown in Fig. 2(b). The isotropic criterion gives a single $\varphi \sim b$ relation for all sections which significantly overestimates the value of friction angle in Sectors II and III. In contrast, the prediction of the anisotropic failure criterion is in good accordance with the test data in Sectors I and II, only slightly overestimates the value of friction angle in Sector III with a maximum difference of 4° at $b = 1$ in sector III, about 10% of the measured friction angle.

3.2 K_0-Consolidated San Francisco Bay Mud (Lade & Kirkgard, 2000)

A series of torsion shear tests have been carried out by Lade & Kirkgard (2000) on K_0-consolidated San Francisco Bay Mud using hollow cylinder torsion shear apparatus. Various stress paths were applied to achieve the full range of stress rotation from $\zeta = 0^\circ$ to $\zeta = 90^\circ$ where ζ is the rotation angle between the direction of major principal stress and the axis of bedding plane of the soil. As there are insufficient test results available in the meridian plane, we assume here $n = 1$ and $\sigma_0 = 0$ kPa for simplicity. M_f is calculated based on the peak friction angle at $b = 0$ ($\varphi_c = 34.1^\circ$), which corresponds to the conventional triaxial compression shear mode. d and β are determined based on the results at $b = 0.5$ and $b = 1.0$ by assuming $\alpha = 0$. Presented in Fig. 3 is the comparison between the test data and the prediction of the anisotropic criterion. The anisotropic failure criterion performs better in the prediction of high b regime than the isotropic one does. It slightly overestimates the values of friction angle when $0.1 < b < 0.4$. Lade & Kirkgard (2000) have remarked that the K_0-consolidated samples of San Francisco bay mud appear to retain the original in-situ fabric which is essentially different from that in the isotropically consolidated remolded specimens tested by Kirkgard & Lade (1993).

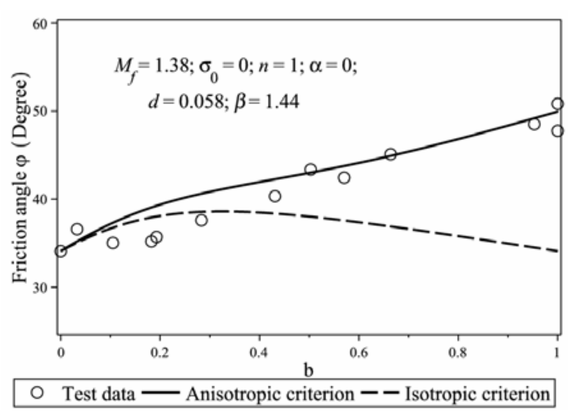
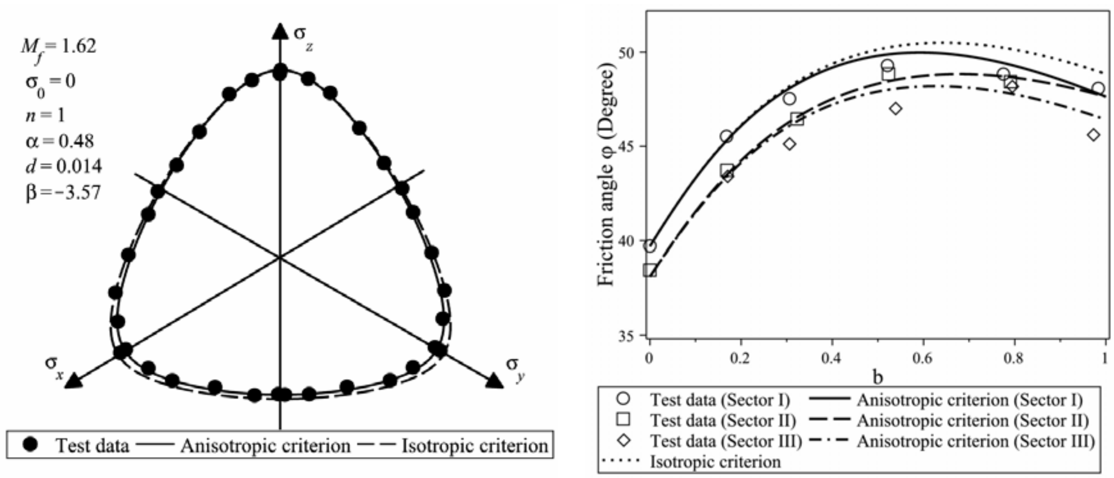


Fig. 3. Comparison between model predictions and torsion shear test results on K_0-consolidated San Francisco Bay Mud (Lade & Kirkgard, 2000) in the *φ-b* diagram.

3.3 Cambria Sand (Ochiai & Lade, 1983)

The anisotropic failure criterion has also been employed to predict the strength of Cambria sand with comparison against the test data obtained by Ochiai & Lade (1983). All the triaxial test results are projected onto the same deviatoric plane with a mean stress $p = 334$ kPa . As shown in Fig. 4(a), for Cambria sand, the effect of anisotropy on the failure curve in the deviatoric plane appears to be relatively small, while the two criteria produce very close predictions. In the $\varphi \sim b$ plane, however, the variation of friction angle with b demonstrates an appreciable dependence on anisotropy in all three sectors, as is shown in Fig. 4(b). The isotropic failure criterion fails to capture this property of Cambia sand. We also notice that the prediction by Lade's anisotropic failure criterion (Lade, 2008) slightly underestimates the φ value between $b = 0.7$ and $b = 1.0$ in all three sectors. Our anisotropic failure criterion captures the trend of $\varphi \sim b$ relation better in both Sector I and Sector II. It only slightly overestimates the value of φ at $b = 0.3$ to $b = 1.0$ in Sector III by about 1°.

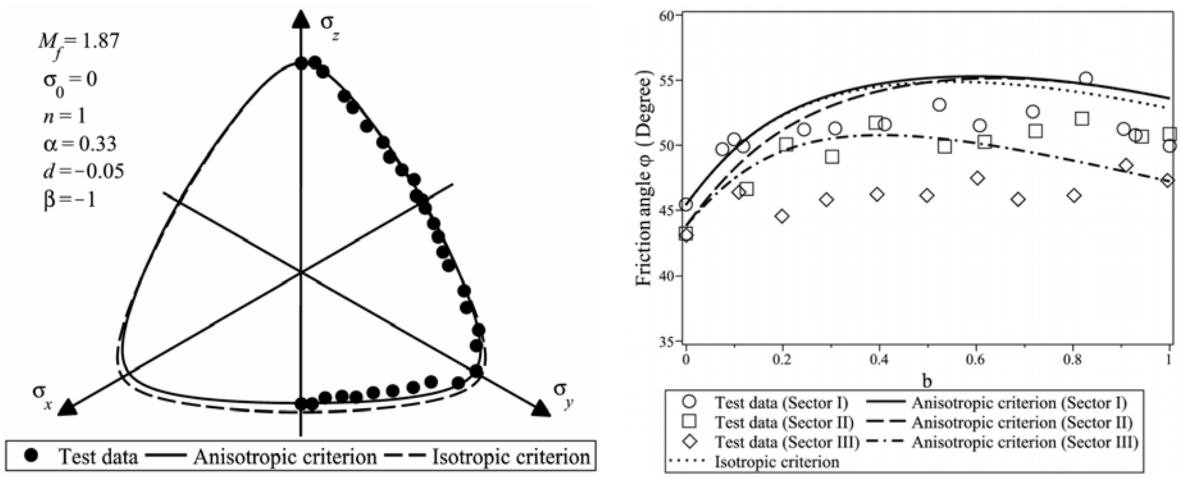


(a) (b)

Fig. 4. Comparisons of the test data on Cambria Sand (Ochiai & Lade, 1983) with predictions by the isotropic and anisotropic failure criteria in (a) the deviatoric plane and (b) the *φ-b* diagram. Variation of the friction angles shows more significant anisotropic effect than that of the strength in the deviatoric plane does.

3.4 Dense Santa Monica Beach Sand (Abelev & Lade, 2004)

True triaxial tests have been carried out by Abelev & Lade (2004) on dense Santa Monica Beach sand deposited with a cross-anisotropic fabric. All tests have been performed with a constant effective cell pressure of $\sigma_3 = 50$ kPa and a constant value of b . Shear banding was observed in the hardening regime in the midrange of b values in each sector of the deviatoric plane. Model predictions by our model and the isotropic criterion are compared to the test data in Fig. 5. As is shown, our anisotropic criterion demonstrates an overall better fitting to the acquired test data than the isotropic failure criterion in both the deviatoric plane and the $\varphi \sim b$ diagram. Nevertheless, we also observe in Fig. 5(b) that the peak friction angle of the sand in the midrange of b values is overestimated by the anisotropic failure criterion in all three sectors. The formation of shear banding may be the attributable to this difference. Indeed, according to Abelev & Lade (2004) and Lade (2007, 2008), occurrence of shear banding may reduce the strength measured from the boundary of the samples. The anisotropic criterion is therefore expected to serve as a target of strength that the material could have attained if the deformation were uniform in the tested sample. We also comment that the prediction by Lade's anisotropic failure criterion (Lade, 2008) in the deviatoric plane is roughly the same as our prediction for this sand; whereas for the $\varphi \sim b$ relation, the prediction by Lade (2008) appears to be slightly better.



(a) (b)

Fig. 5. Failure of dense Santa Monica Beach sand predicted by the isotropic and anisotropic failure criteria in comparison with test data (Abelev & Lade, 2004), in (a) the deviatoric plane and (b) φ-b diagram in three sectors. The anisotropic criterion overestimates the strength in the midrange of b due to shear banding.

3.5 Toyoura Sand (Lam & Tatsuoka, 1988)

Lam & Tatsuoka (1988) carried out true triaxial tests on Toyoura sand with a constant cell pressure of $\sigma_3 = 98$ kPa where the sand samples have been prepared by the air-pluviating method to introduce initial cross-anisotropic fabric. Shear banding has been observed in their testing. The predicted strength by our model is compared to the test data in Fig. 6(a). In the deviatoric plane, the anisotropic failure criterion provides a better correlation with the test data than the isotropic criterion does. It does, however, slightly overestimate the strength in the midrange of b values in Sector II and Section III, which is similar to the case of Santa Monica Beach sand.

Tatsuoka et al. (1990) have later carried out triaxial compression tests on Toyoura sand for which it is also interesting to make a comparison with our model prediction. Presented in Fig. 6(b) is the variation of the friction angle with the loading direction in term of ξ (the angle between the direction of major principal stress and the axis of bedding plane of the soil, same as the ζ in the rotoational shear case) at a constant confining pressure of 98 kPa obtained by the triaxial compression tests (Tatsuoka et al., 1990), in close comparison with the predictions by the isotropic and anisotropic failure criteria. Note that two set of data are presented in the figure which correspond to samples with different initial void ratios. Since the strength of sand is known to be affected by the initial void ratio as well as confining pressure, the data in Fig. 6(b) have been normalized by the friction angle at $\xi = 0^\circ$ (denoted as φ_0) for consistency. No shear banding has been observed in the triaxial compression tests. As is shown in Fig. 6(b), the prediction by the anisotropic failure criterion for the triaxial tests on Toyoura sand compares favorably with the test data, whilst the constant prediction by the isotropic criterion deviates from the test data by a large extent when ξ becomes greater.

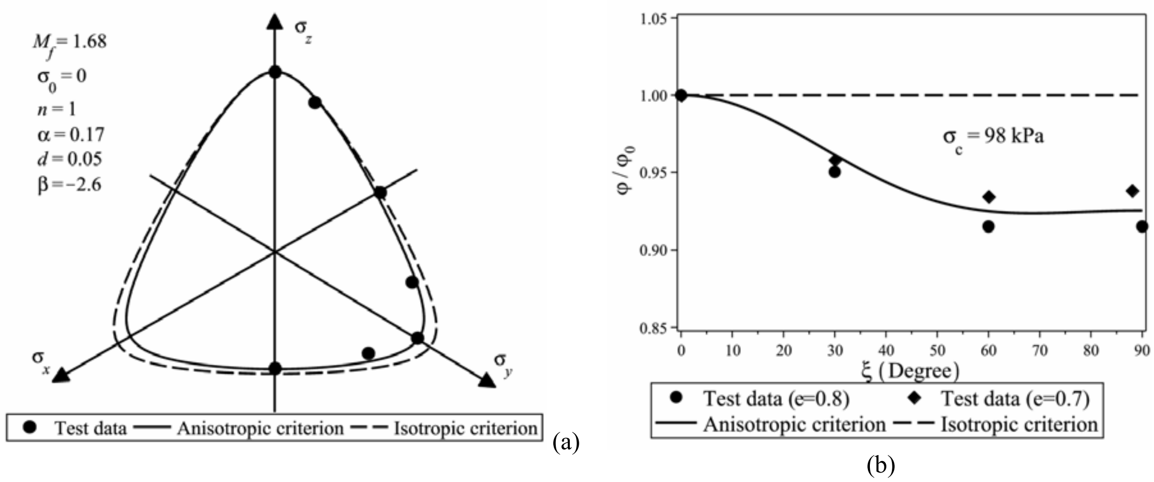


Fig. 6. Prediction of the strength of Toyoura Sand by the isotropic and anisotropic failure criteria in comparison with (a) the true triaxial test results by Lam & Tatsuoka (1988) in the deviatoric plane; and (b) the triaxial compression test results by Tatsuoka et al. (1990) in the $\varphi / \varphi_0 - \xi$ diagram.

3.6 Santa Monica Beach Sand (Lade et al., 2008)

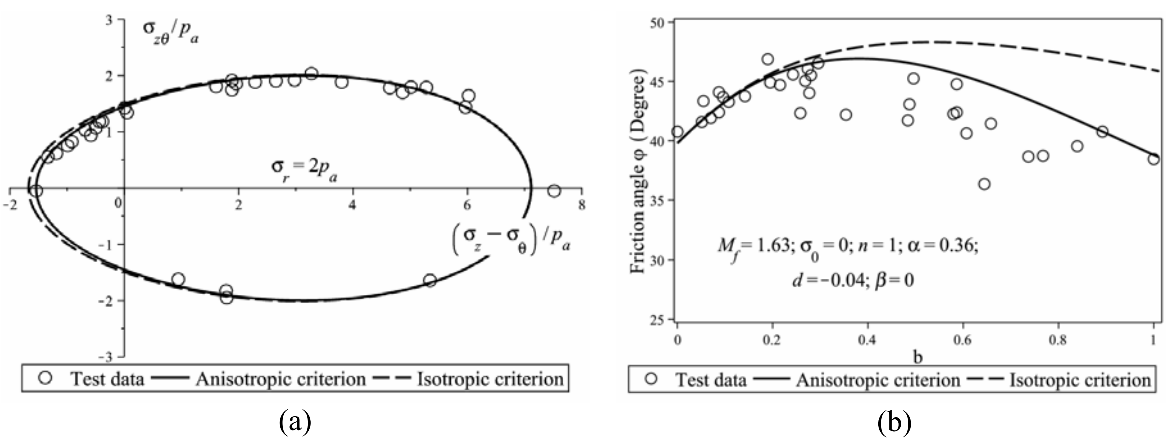


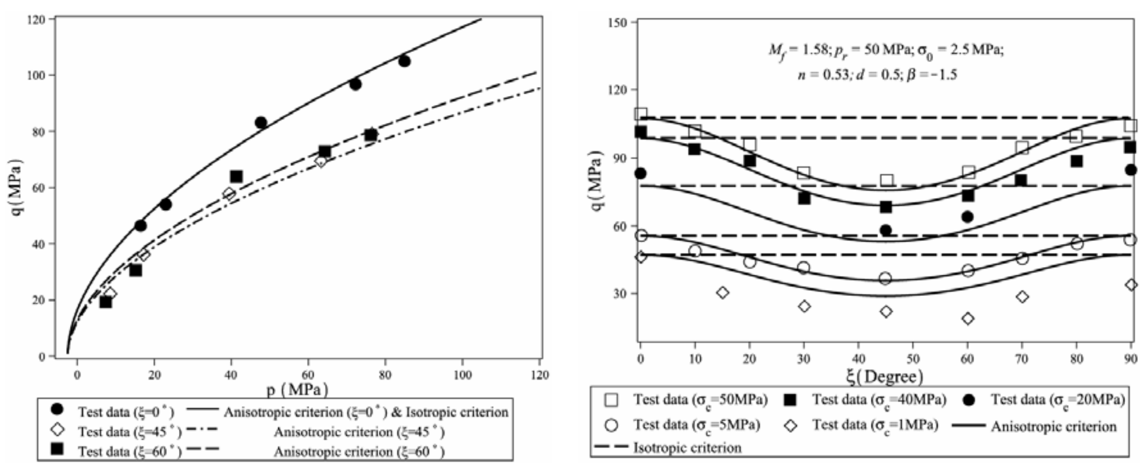
Fig. 7. Comparison between the torsion shear test results on dry-pluviated Santa Monica Beach sand (Lade et al., 2008) with the predictions by the isotropic/anisotropic failure criteria in (a) the $\sigma_{z\theta} \sim (\sigma_z - \sigma_\theta)$ diagram, and (b) the *φ*-*b* diagram.

A total of 34 torsion shear tests have been carried out by Lade et al. (2008) on dry-pluviated Santa Monica Beach sand. The tests were conducted under drained conditions at a cell pressure of 200 kPa applied to both the inner and outside cell walls. For comparison with the torsion shear tests, 11 true triaxial tests have also performed in the cubic triaxial apparatus with four different confining pressures. Since the curvature of the failure curve in the meridian plane can not be determined based on the available test data, n is set to be unity here. M_f is calculated based on the average value of the friction angles obtained in the conventional triaxial compression tests performed in the torsion shear apparatus and the cubic triaxial apparatus respectively ($\varphi_c = 34.1^\circ$). The friction angle obtained in true triaxial tests at

$b=1$ ($\varphi_e = 46^\circ$), which corresponds to the shear modes of $\theta = 60^\circ$ and $\xi = 0^\circ$, is used to determine the parameter α. Shear banding has been observed in most of the torsion shear tests by Lade et al. (2008). To minimize the influence of shear banding, we only select the results obtained at $b=1$, which corresponds to the shear mode of $\theta = 180^\circ$ and $\xi = 0^\circ$, for determining the parameter d by setting $\beta = 0$. The predictions are presented in Fig. 7. While both the isotropic criterion and the anisotropic criterion capture the test data reasonably well in the plane of $\left(\sigma_z - \sigma_\theta\right) \sim \sigma_{z\theta}$ in Fig. 7(a), it is in the $\varphi \sim b$ plane that the difference can be better depicted. As shown in Fig. 7(b), the isotropic failure criterion clearly overestimates the measured strength when $b > 0.3$. The anisotropic failure criterion, on the other hand, can capture the overall trend of strength variation with b better. Noticeable overestimation, though, is still observed in the range of $0.3 < b < 0.85$ where shear banding comes into effect.

3.7 Touremire Shale (Niandou et al., 1997)

A comparison has also been made between predictions by our anisotropic failure criterion with the triaxial test data on the Touremire shale in Niandou et al. (1997), which is shown in Fig. 8. The prediction by the isotropic criterion is also shown for the convenience of comparison. As is seen in Fig. 8(a), the anisotropic failure criterion satisfactorily captures the $p \sim q$ relation at different loading directions for Touremire shale. Its predictions also agree well with the test data at most confining pressure levels, only with a slight overestimation on the strength at a low confining pressure of $\sigma_c = 1$ MPa when $\xi > 0^\circ$ and a moderate underestimation for the case of $\sigma_c = 20$ MPa (Fig. 8(b)). The observed deviation may be possibly due to the fact that the anisotropic variable A introduced in this paper is assumed to be only a measure of the stress direction relative to the material fabric orientation but independent on the mean stress. According to Niandou et al. (1997), the degree of strength anisotropy is greater at lower confining pressure levels than that at higher ones. This pressure-dependent strength anisotropy has also been observed by Lade & Abelev (2005) in sand. A potential improvement of the current anisotropic failure criterion may be done by incorporating the effect of mean stress in the criterion, e.g., via the anisotropic variable A.

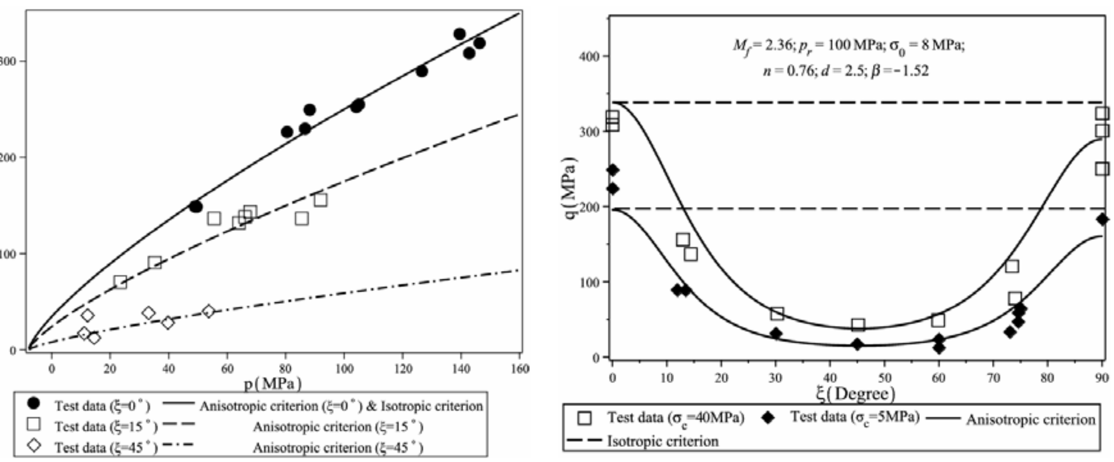


(a) (b)

Fig. 8. Comparison between the triaxial compression test data on the Touremire shale (Niandou et al., 1997) and the prediction of the anisotropic and underlying isotropic failure criteria in (a) the *p-q* diagram with different loading directions and (b) the *ξ-q* diagram with different confining pressures.

3.8 Angers Schist (Duveau et al., 1998)

Another rock, the Angers schist reported in Duveau et al. (1998), has been used to validate our failure criterion. The test data for Angers schist are more scattered compared to those for the Touremire shale (Fig. 9). Our anisotropic failure criterion, with the chosen parameters, can reasonably capture the overall trend of the data set in both the $p \sim q$ and $\xi \sim q$ planes. In the $\xi \sim q$ plane as shown in Fig. 9(b), the criterion slightly overestimates the strength at all range of ξ except $\xi = 0^\circ$ and 90°. The isotropic failure criterion fails to capture the strength variation with loading directions for both rocks in either the $p \sim q$ plane or the $\xi \sim q$ plane.



(a) (b)

Fig. 9. Comparisons of the triaxial compression test data on the Angers schist (Duveau et al., 1998) with the prediction of the anisotropic and underlying isotropic failure criteria in (a) the *p-q* diagram with different loading directions and (b) the *ξ-q* diagram with different confining pressures.

4 CONCLUSION AND DISCUSSION

The new anisotropic failure criterion presented in this paper has been demonstrated to be general and robust enough to provide excellent predictions on the strength anisotropy for a wide range of soils and rocks. The introduction of the anisotropic variable A in terms of the invariants and joint invariants of the stress tensor and the fabric tensor appears to be effective in characterizing the effect of fabric anisotropy. Parameters introduced in the criterion can be conveniently calibrated by conventional laboratory tests. We note that the usefulness of this criterion is not limited to geomaterials only. For any materials that exhibit appreciable strength anisotropy, such as *concrete*, *ceramics*, *porous metals*, *polymers* and *solid metals*, it can be equally useful. The specific procedures in determining the required parameters may differ from those mentioned in this paper, though, depending on the availability of routine tests for these different materials. Meanwhile, we have mentioned that shear banding has been observed in the hardening regime in true triaxial tests on sand (Wang & Lade, 2001; Abelev & Lade, 2003) in the midrange of b values (from about 0.18 to approximately 0.85). The occurrence of shear banding in the hardening regime prevents the attainment of a smooth peak on the stress-strain relation for which a failure criterion tends to fit with. For the range of b where shear banding occurs, the current anisotropic failure criterion slightly overestimates the strength of the soils in the deviatoric plane. In this case, the prediction of strength by our criterion can be regarded as a targeted upper bound that a soil can achieve if the deformation is uniform in the tested sample. In addition, the anisotropic failure criterion presented here can be easily extended for constitutive modeling of geomaterials as well, by simply considering it as the yield function with suitable hardening rules specified, as is the way done by Pietruszczak et al. (2002) and Azami et al. (2009). It is also possible to introduce a cap in the meridian plane for the proposed failure criterion for various useful purposes in constitutive soil modeling.

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