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RESEARCH PAPER

A failure criterion for cross-anisotropic soils considering microstructure

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Abstract Strength anisotropy is an important soil behavior which is closely related to the microstructural characteristics in soil. Proper consideration of the microscopic properties of a soil is key to accurate description of its strength anisotropy. This paper presents a failure criterion for cross-anisotropic soils based on the Spatially Mobilized Plane (SMP) criterion. In the new criterion, the shear-normal stress ratio defined in the SMP is related to the relative orientation of the loading direction with respect to the axes of material anisotropy through a microstructure-based tensor. The formulation includes only two parameters which can be easily calibrated by conventional triaxial tests. By comparison with tests data, the criterion is shown to offer good characterization of the strength anisotropy for a variety of geomaterials with cross-anisotropy.

Keywords Anisotropy · Failure criterion · Microstructure · Shear strength · Soil and rock

1 Introduction

Due to depositional process and other microstructural patterns, naturally occurring materials, such as soil and rock, are typically cross-anisotropic [2]. The material behavior within the depositional plane is found largely isotropic, while it is quite different in the direction perpendicular to this plane (the direction is commonly called *the axis of anisotropy*). The cross-anisotropic structure of geomaterials affects many facets of the material behavior. One outstanding example is the strong dependence of shear strength on the loading direction relative to the symmetric plane of a soil or rock. This phenomenon cannot be satisfactorily described by isotropic failure criteria, such as the Mohr–Coulomb criterion, the Drucker–Prager criterion, the Hoek–Brown criterion, the SMP criterion [12] or Lade's isotropic failure criterion [7]. Proper characterization of the strength anisotropy for geomaterials has been an active research area in geomechanics, driven partly by its obvious importance to practical design in geotechnical engineering.

In describing the phenomenon of different strengths observed in different loading conditions for the same soil, conventional approaches usually stick to an isotropic failure criterion (e.g., the Lade's criterion [7]) but adopt different friction angles. While the friction angle has long been considered as an intrinsic material property, changing the friction angles for the same soil at different loading conditions is apparently questionable. As highlighted by numerous past studies, it is indeed the presence of material anisotropy that gives rise to the observed strength discrepancy in different loading conditions. Consequently, there have been a variety of failure criteria developed for geomaterials based on different approaches in consideration of the material anisotropy with respect to the loading direction [1, 4, 8, 11, 17, 18]. For example, using a coordinate rotation of the principal stress space and utilization of Lade's isotropic failure criterion [7], a three-dimensional failure criterion for cross-anisotropic soils was proposed by

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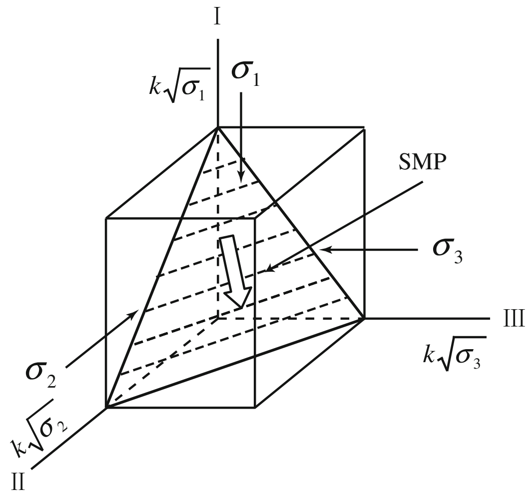


Fig. 1 Spatially mobilized plane in the principal stress space [14]

Abelev and Lade [1]. An anisotropic variable defined by the joint invariant of a second-order fabric tensor and the stress tensor has been employed in [4] in their formulation. Pietruszczak and Mroz [17, 18] have proposed a microstructure-based tensor in developing their failure criterion for rocks. This microstructure tensor has been further adopted by Lade [8] in conjunction with his own isotropic failure criterion to formulate a new one for cross-anisotropic soils. Liu and Indraratna [11] have employed a vector to represent the anisotropy of material strength. These criteria have gained varied degree of success in predicting the strength anisotropy for various materials. The major drawbacks associated with the existing studies, however, lie in the large number of material parameters (or functions) required to be identified in their formulations. In many cases, advanced tests, such as true triaxial testing, are needed to calibrated these parameters, which is indeed inconvenient for practical use. Importantly, microstructure has been recognized attribute to important material behavior of geomaterials on the continuum level such as strength anisotropy. Most existing failure criteria involve parameters and failure mechanisms which are ambiguously correlated to the microstructural characteristics of the material [17].

This paper aims to develop a new general failure criterion for transversely isotropic geomaterials based on consideration of microstructure. We adopt the isotropic SMP criterion [12] to quantify the mobilized strength in a frictional material. The SMP criterion features a clear microstructure-based failure mechanism of spatial mobilized plane (see Fig. 1) along which sliding of soil particles is believed to take place to the greatest extent in the principal stress space. Meanwhile, the microstructure tensor-based approach proposed in [17, 18] will be employed to consider the coupled effect of loading direction and material anisotropy. The combination of the two approaches will lead to a new three-dimensional failure criterion featuring not only a clear microstructural concept for the failure of transversely isotropic materials, but also with only two material parameters which can be easily calibrated by conventional triaxial tests. The new failure criterion offers reasonable predictive capability on describing the strength anisotropy for a wide range of materials, as will be demonstrated in the subsequent sections.

2 Formulation of the anisotropic failure criterion

It is instructive to briefly introduce the microstructure tensor-based approach by Pietruszczak and Mroz [17] first. Centered to their approach is the definition of a scalar anisotropy parameter η though the projection of a microstructure tensor onto the direction of the generalized loading direction.

The microstructure tensor a_{ij} used by Pietruszczak and Mroz [17] denotes a tensorial measure of the material fabric describing such microstructural characteristics as the spacial distribution of particles, voids or the arrangement of inter-granular contacts in a material. It indeed shares a similarity with the fabric tensor defined by Oda [15, 16] which was used in [4]. The principal triad of a_{ij} is expressed by the unit vectors v_i, s_i, t_i (as shown in Fig. 2), and the spectral representation of a_{ij} [17]

$$a_{ij} = a_1 v_i v_j + a_2 s_i s_j + a_3 t_i t_j \tag{1}$$

where a_1, a_2, a_3 are the principal values of the microstructure tensor. For convenience of manipulation, the microstructure tensor a_{ij} is often defined to be coaxial with the axes of orthotropy of the material.

It is necessary to specify the loading orientation with respect to the microstructure directions of the material before a failure criterion for anisotropic geomaterials can be formulated. Figure 2 shows the components of loading orientation relative to the axes of the cross-anisotropic material microstructure. The magnitudes of the resultant stresses acting on the planes characterized by normals 1, 2 and 3 are calculated as

$$L_1 = \sqrt{(\sigma_{11}^2 + \sigma_{12}^2 + \sigma_{13}^2)} \tag{2}$$

$$L_2 = \sqrt{(\sigma_{12}^2 + \sigma_{22}^2 + \sigma_{23}^2)} \tag{3}$$

$$L_3 = \sqrt{(\sigma_{13}^2 + \sigma_{23}^2 + \sigma_{33}^2)} \tag{4}$$

The following generalized loading vectors are defined [17]

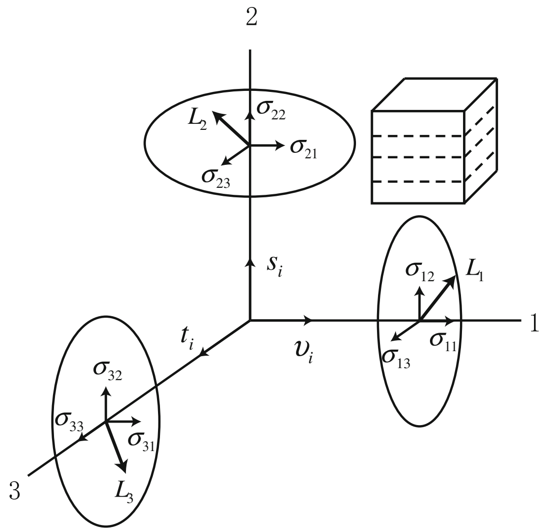


Fig. 2 Description of the load orientation relative to axes of the cross-anisotropic material microstructure

$$L_i = L_1 v_i + L_2 s_i + L_3 t_i \quad (i = 1, 2, 3) \tag{5}$$

in which v_i, s_i and t_i are unit vectors as shown in Fig. 2. The unit vector specifying the loading direction is expressed as

$$(l_1, l_2, l_3) = \frac{(L_1, L_2, L_3)}{\sqrt{L_1^2 + L_2^2 + L_3^2}} \tag{6}$$

With the generalized loading direction defined above, Pietruszczak and Mroz [17, 18] proposed the following formulation of failure criteria for anisotropic materials

$$f = \eta = \eta_0(1 + \Omega_{ij} l_i l_j) \tag{7}$$

Here, f is the function of stresses. η is a scalar parameter specifying the effect of load orientation l_i with respect to the material axes and is a homogeneous function of stress of degree zero, such that it does not depend on the magnitude of stresses, but only on the relative orientation of the principal stress triad with respect to the eigenvectors of the microstructure tensor. In Eq. (7), η_0 is the average value of η, which is defined as follows and denotes the radius of the sphere as shown in Fig. 3

$$\eta_0 = \frac{a_{kk}}{3} \tag{8}$$

In Eq. (7) Ω_{ij} is a deviatoric measure of the material microstructure [17] defined below

$$\Omega_{ij} = \frac{a_{ij} - \frac{1}{3}\delta_{ij} a_{kk}}{\frac{1}{3} a_{kk}} \tag{9}$$

where δ_{ij} is the Kronecker delta. For an orthotropic material, there are two distinct eigenvalues for Ω_{ij}. As such, only one scalar chosen from either of the two eigenvalues of Ω_{ij} (e.g., Ω_1) is sufficient to characterize the cross-anisotropy of the material. Evidently, for an isotropic material $\Omega_{ij} = 0$, which will be discussed in detail in the following section.

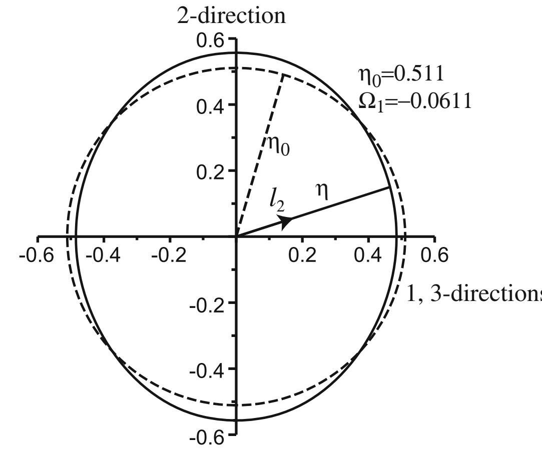


Fig. 3 Variation of η indicated by the symmetric shape for San Francisco Bay Mud

In essence, Eq. (7) embodies an perturbation of the material anisotropy by its relative orientation with respect to the loading direction used in defining a failure criterion.

3 A new failure criterion for cross-anisotropic soils

In general cases, Eq. (7) can be recast as

$$\eta = \eta_0(1 + \Omega_1 l_1^2 + \Omega_2 l_2^2 + \Omega_3 l_3^2) \tag{10}$$

For cross-anisotropic materials, given that $\Omega_1 = \Omega_3$, $\Omega_1 + \Omega_2 + \Omega_3 = 0$ and $l_1^2 + l_2^2 + l_3^2 = 1$, then Eq. (10) becomes

$$\eta = \eta_0[1 + \Omega_1(1 - 3l_2^2)] \tag{11}$$

As such, only two parameters, η_0 and Ω_1 need to be determined, while the loading direction l_2 can be determined from the stress state.

As mentioned in the introduction, the SMP failure criterion [12] offers the critical shear-normal stress ratio (τ/σ_N) on a spatial mobilized plane (Fig. 1) along which failure of an isotropic soil may likely occur. The stress ratio can be expressed in terms of stress invariants as

$$\frac{\tau}{\sigma_N} = \sqrt{\frac{I_1 I_2}{9 I_3} - 1} \tag{12}$$

in which I_1, I_2 and I_3 are respectively the first, second and third stress invariants of the stress tensor

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{13}$$

$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \tag{14}$$

$$I_3 = \sigma_1\sigma_2\sigma_3 \tag{15}$$

Here, σ_1, σ_2 and σ_3 are principal effective stress components.

The isotropic SMP failure criterion exhibits a curved failure locus circumscribing the Mohr–Coulomb hexagon in the deviatoric plane. It was originally developed for cohesionless granular materials only. Matsuoka et al. [13] have further extended it for cohesive-frictional materials, by introducing the following bonding stress

$$\sigma_0 = c\cot\varphi \quad (16)$$

where c is the cohesion, and φ is the internal friction angle. Based on σ_0, the following translated principal stress $\hat{\sigma}_i$ and translated stress invariants $\hat{I}_1$, $\hat{I}_2$, $\hat{I}_3$ are defined

$$\hat{\sigma}_i = \sigma_1 + \sigma_0 \quad (i = 1, 2, 3) \quad (17)$$

$$\hat{I}_1 = \hat{\sigma}_1 + \hat{\sigma}_2 + \hat{\sigma}_3 \quad (18)$$

$$\hat{I}_2 = \hat{\sigma}_1\hat{\sigma}_2 + \hat{\sigma}_2\hat{\sigma}_3 + \hat{\sigma}_3\hat{\sigma}_1 \quad (19)$$

$$\hat{I}_3 = \hat{\sigma}_1\hat{\sigma}_2\hat{\sigma}_3 \quad (20)$$

Using these translated stress invariants in place of the original stress invariants in Eq. (12), the shear strength of cohesive-frictional materials can also be characterized.

In this paper, we proposed the following failure criterion for cross-anisotropic materials by "marrying" the SMP criterion with the η proposed by Pietruszczak and Mroz [17, 18]:

$$\frac{\tau}{\sigma_N} = \sqrt{\frac{I_1 I_2}{9I_3} - 1} = \eta = \eta_0[1 + \Omega_1(1 - 3l_2^2)] \quad (21)$$

For general cohesive-frictional materials, the shear-normal stress ratio can be calculated by using the translated stress invariants defined above. The values of parameters η_0 and Ω_1 are considered to be microstructure related soil parameters. η_0 is the average value of η, which represents the maximum value of shear-normal stress ratio on a spatial mobilized plane of a soil under the loading direction defined by l_2. Detailed discussion on l_2 will be given in the following section.

Physically, the criterion in Eq. (21) may be interpreted as a negotiation between the possible failure plane defined by SMP and the material bedding plane. If the spatial mobilized plane is closer to the perpendicular plane of the bedding plane, it is likely induced a higher shear strength. Otherwise, a smaller shear strength will be observed. Mathematically, we note that the expression in Eq. (21) resembles the form used by Lade [8] who also used η. While we emphasize a clear microstructural concept in this study, the new criterion has only two parameters to be calibrated, one less than used by Lade [8]. Nevertheless, it is noteworthy that strength anisotropy may be stress or density dependent. Generally, it was observed that the behavior of anisotropy decreases with increased isotropic pressure [9]. Lade's [8] criterion has been able to account for this effect with the extra parameter. However, at the common stress range soils are experienced or tested in practice, the effect can be neglected.

4 Specific expressions of the loading direction

Following Lade [8], we partition the deviatoric plane into three sectors as shown in Fig. 4. Also shown in the figure are the different stress states applied to a horizontally bedded cross-anisotropic soil sample in each sector. It is expedient to use the intermediate principal stress ratio, $b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$, to express the stress state. Evidently, $b = 0$ at triaxial compression (TC) and $b = 1$ at triaxial extension (TE). Note also in this figure, x-y plane coincides with the bedding planes of the sample, and z-axis is the axis of orthotropy.

In soil mechanics, the following definition of major stress ratio R is commonly used [20] (note that a similar definition may be used for the translated principal stresses defined in previous section)

$$R = \frac{\sigma_1}{\sigma_3} \quad (22)$$

The cross-anisotropic failure criterion can then be expressed in terms of R and b as follows

$$\frac{\tau}{\sigma_N} = \sqrt{\frac{(bR + R + 2 - b)(bR^2 + 2R + 1 - b)}{9R(bR + 1 - b)} - 1} = \eta \quad (23)$$

To calculate the right-hand side of Eq. (21), it is convenient to give the specific expressions of l_2^2 for different loading

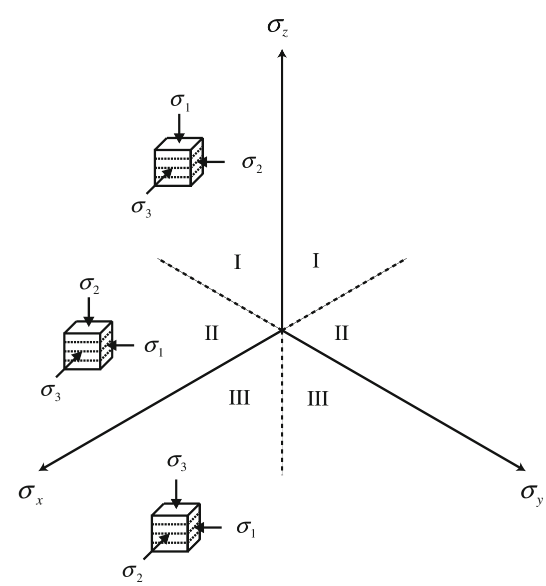


Fig. 4 Orientation of the soil samples showing cross-anisotropy in all three sectors in the π-plane[8]

conditions. Special loading conditions that are commonly taken for true triaxial tests and torsional shear tests are considered, by following the procedure outlined by Lade [8].

4.1 True triaxial tests

Under true triaxial conditions as shown in Fig. 4, the principal stresses coincide with the axis of orthotropy. As a result, the following specific expressions for l_2^2 in the three sectors can be obtained.

In Sector I:

$$l_2^2 = \frac{R^2}{R^2 + [b(R-1)+1]^2 + 1} \quad (24)$$

In Sector II[1]:

$$l_2^2 = \frac{[b(R-1)+1]^2}{R^2 + [b(R-1)+1]^2 + 1} \quad (25)$$

In Sector III:

$$l_2^2 = \frac{1}{R^2 + [b(R-1)+1]^2 + 1} \quad (26)$$

4.2 Torsional shear tests

Under a typical torsional shear test, the stress condition is shown in Fig. 5 respect to the bedding plane of the sample. The inside and outside pressures are maintain at the same value of σ_r, which is equal to the intermediate principal stress σ_2. The orientation of the major principal stress to the axis of anisotropy is β, and it is related to the value of b according to $b = \sin^2\beta$ [8, 10]. As a result, the expression of l_2^2 can be obtained as

$$l_2^2 = \frac{R^2(1-b)+b}{R^2 + [b(R-1)+1]^2 + 1} \quad (27)$$

As such, for $b = 0$, $l_2^2 = R^2/(R^2+2)$, and for $b = 1$, $l_2^2 = 1/(2R^2+1)$.

5 Parameter calibration

The new criterion needs to determine two material parameters, η_0 and Ω_1. By far conventional triaxial compression/extension tests remain the major testing types that are routinely accessible to practicing engineers. It is hence more convenient if a failure criterion can be calibrated by these routine tests. Reliable parameters for the failure criterion can be obtained from different sets of test data that show the cross-anisotropic features between them. Indeed, test results can be obtained on horizontal and vertical samples of cross-anisotropic soils as shown in Fig. 6, from which the two parameters

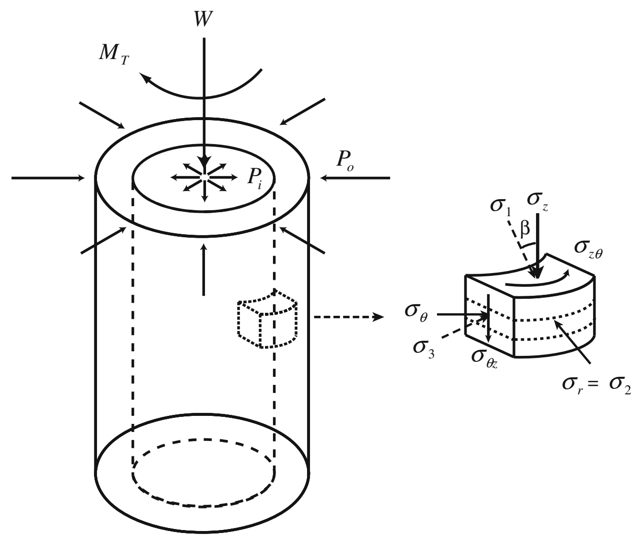


Fig. 5 Stress state in torsional shear tests

can be easily determined. If torsional shear tests are available, they can also be used for the parameter calibration. A systematic procedure on the parameter determination is given as follows. As mentioned in the Introduction, these parameters should be material constants independent of the specific calibration tests, rather than variable quantities as the conventional ways manipulating the friction angle.

5.1 From triaxial tests

We take the case of San Francisco Bay Mud [6] as an demonstrative example here. Conventional triaxial compression tests on both vertical samples and horizontal samples have been conducted for this soil. The friction angle obtained from tests on the vertical sample (Fig. 6a) is $\varphi_v = 30.6°$, while for the horizontal sample case (Fig. 6b) it is $\varphi_h = 27.4°$. Under conventional triaxial compression conditions, the principal stress ratio R_f [21] at failure can be calculated from:

$$R_f = \frac{1+\sin\varphi}{1-\sin\varphi} \quad (28)$$

Thus, for the vertical sample $R_{fv} = 3.07$, while for the horizontal sample $R_{fh} = 2.71$. Inserting $b = 0$ under conventional compression condition into Eq. (23) with the two values of R, we have $\eta_v = 0.557$ and $\eta_h = 0.490$. The following two linear equations are readily obtained from Eq. (21).

$$\begin{cases} \eta_v = \eta_0[1 + \Omega_1(1 - 3l_2^2)] \\ \eta_h = \eta_0[1 + \Omega_1(1 - 3l_2^2)] \end{cases} \quad (29)$$

In these two equations, the values of l_2^2 are obtained from Eq. (24) for the vertical sample and from Eq. (26) for the horizontal sample. The two linear equations are then collectively solved for η_0 and Ω_1. For the case of San Francisco Bay Mud, we have $\eta_0 = 0.511$ and $\Omega_1 = -0.0611$.

[1] Eq. (12a) in [8] appears to have been mis-printed with a missing square for R in the numerator of l_2^2.

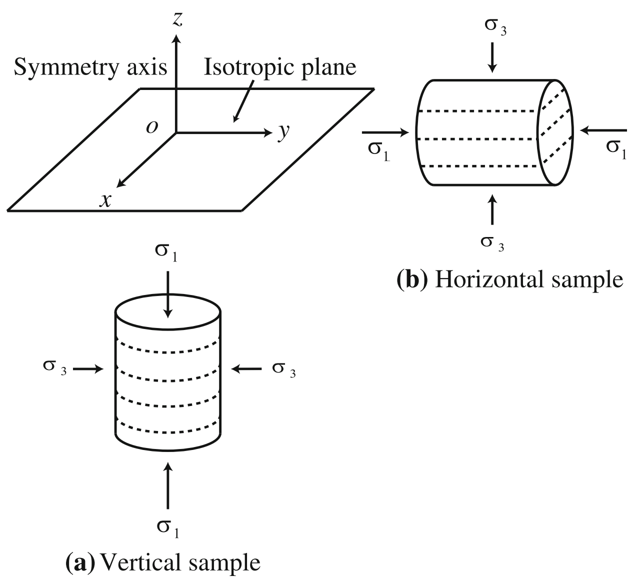


Fig. 6 Vertical and horizontal samples of cross-anisotropic soil

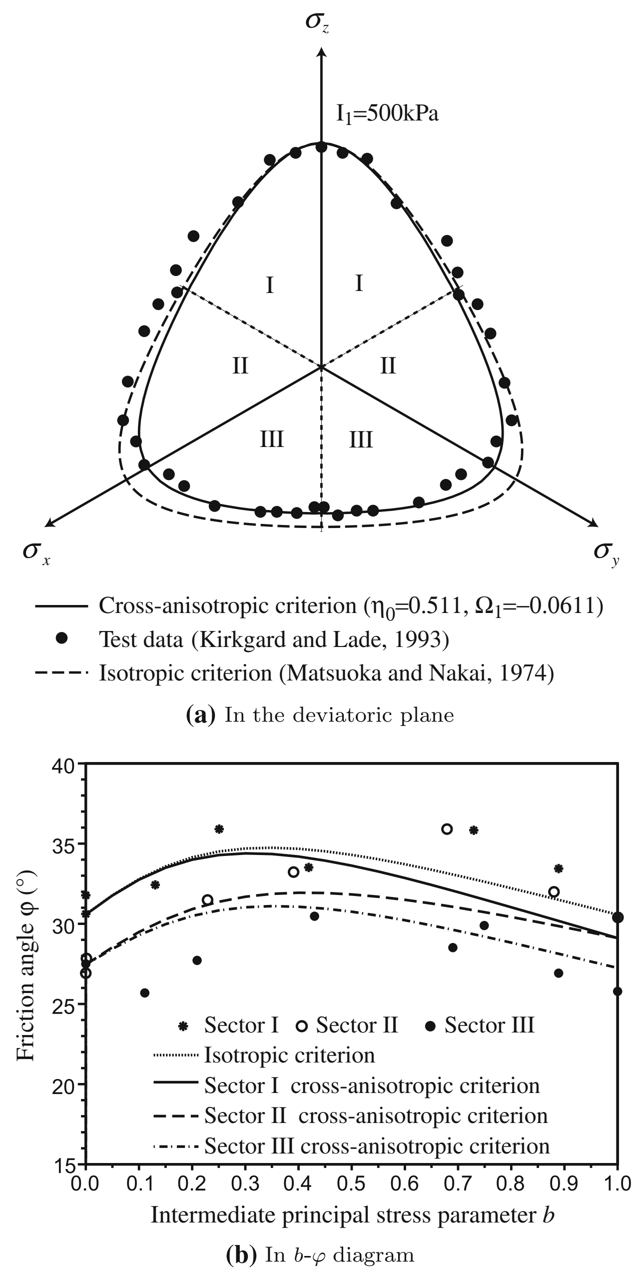


Fig. 7 Comparison of the predictions of the new failure criterion (as well as isotropic SMP criterion) wtih test data for San Francisco Bay Mud [6] (**a**)in the π-plane and (**b**) in $b-\varphi$ diagram

5.2 From torsional shear tests

Tests on Santa Monica beach sand by Lade et al. [10] are taken as an example for this case. A total of 34 drained torsional shear tests were performed with constant effective confining pressure, $\sigma_r{}' = 196$ kPa, for medium dense Santa Monica beach sand ($e = 0.68$ corresponding to a relative density of 70 %). The friction angle under triaxial compression is obtained at $\varphi_c = 40.7°$, while the friction angle under triaxial extension is $\varphi_e = 38.4°$.

With $b = 0$, a value for $R_{fc} = 4.77$ is readily obtained from Eq. (28). Hence from Eq. (23), we have $\eta_c = 0.811$ in this case. For $b = 1$, $R_{fe} = 4.28$ from Eq. (28) and $\eta_e = 0.747$ from Eq. (23). Similar to the case in last subsection, two linear equations are obtained by substituting η_c and η_e into the right-hand side of Eq. (21). With the corresponding values of l_2^2 being calculated from Eq. (27), we can arrive in the following values for the two unknowns for medium dense Santa Monica beach sand: $\eta_0 = 0.769$ and $\Omega_1 = -0.0309$. According to the same procedure, the following pair of parameters have been obtained for the glass beads tested in [5]: $\eta_0 = 0.466$ and $\Omega_1 = -0.0734$.

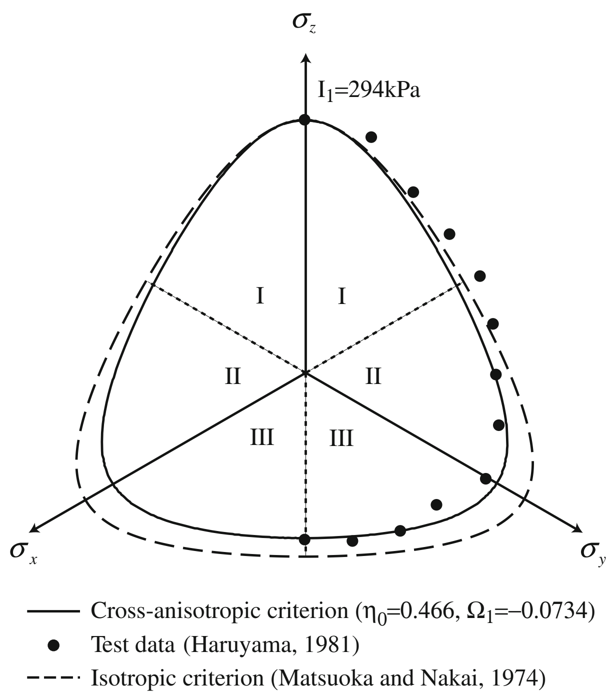
It will be useful to have an approximate range of the two material parameters involved in the proposed criterion for future study as well as practical use. Based on our experience gained from the parametric calibrations, the following ranges are recommended for cross-anisotropic soils: (a) for clay with a friction angle varying from 15° to 40°, the typical ranges are as follows: $\eta_0 \in (0.19, 0.71)$, $\Omega_1 \in (-0.074, -0.450)$; (b) for sand with a friction angle in the range of 25° to 45° the ranges for the two parameters are as follows: $\eta_0 \in (0.37, 0.84)$, $\Omega_1 \in (-0.067, -0.146)$.

6 Evaluation of the proposed criterion against test data

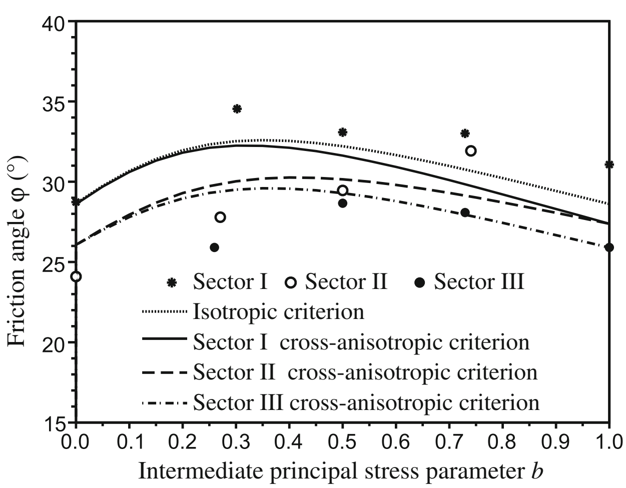
6.1 Triaxial tests

6.1.1 San Francisco Bay Mud [6]

With the parameters determined above, the new failure criterion has been employed to predict the shear strength of San Francisco Bay Mud. A comparison of experimental data with the predictions is presented in Fig. 7. Also



(a) In the deviatoric plane



(b) b-φ diagram

Fig. 8 Comparison of test data for glass beads (Haruyama, 1981[5]) with predictions from the new cross-anisotropic failure criterion as well as isotropic SMP criteria (**a**) in the π-plane and (**b**) in $b-\varphi$ diagram

presented in the figure are the predictions by the isotropic SMP criterion. From both the deviatoric plane in Fig. 7a and the $b-\varphi$ diagram in Fig. 7b, the anisotropic failure criterion clear shows a better comparison with the test data than the isotropic criterion. Notably from Fig. 7a, however, the new failure criterion slightly underestimates the strength of the soil in Section II of the deviatoric plane. In the $b-\varphi$ diagram, its predictions overestimate the shear strength in the low b case for Section III but show a mild underestimation in the high b range of Section I.

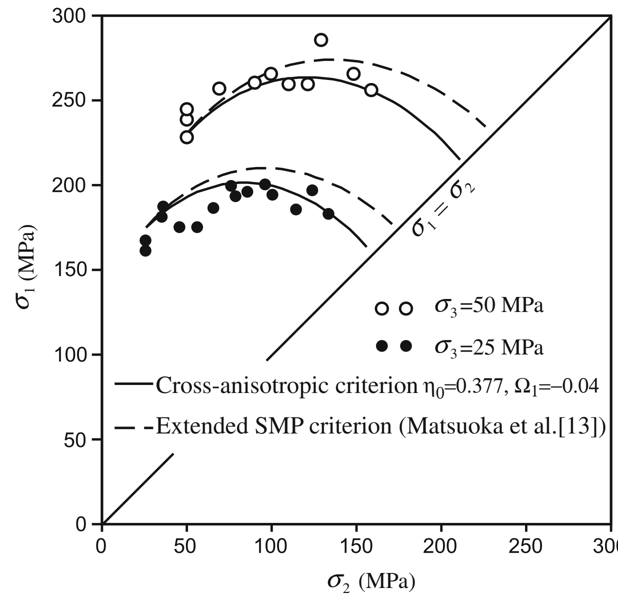


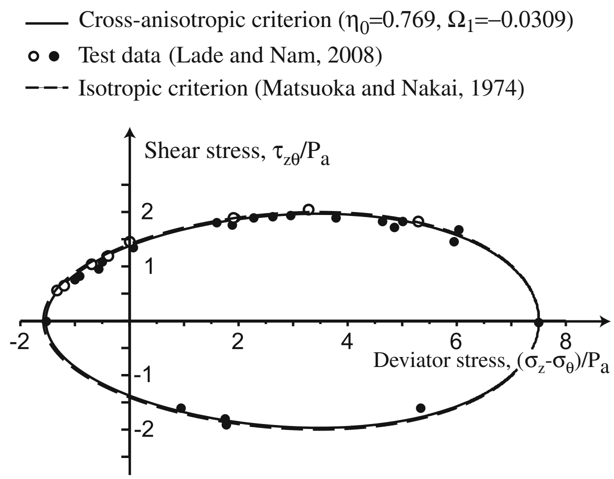
Fig. 9 Comparison of failure criteria with test data for the Yuubari shale (Data after Takahashi [19])

6.1.2 *Spherical glass beads [5]*

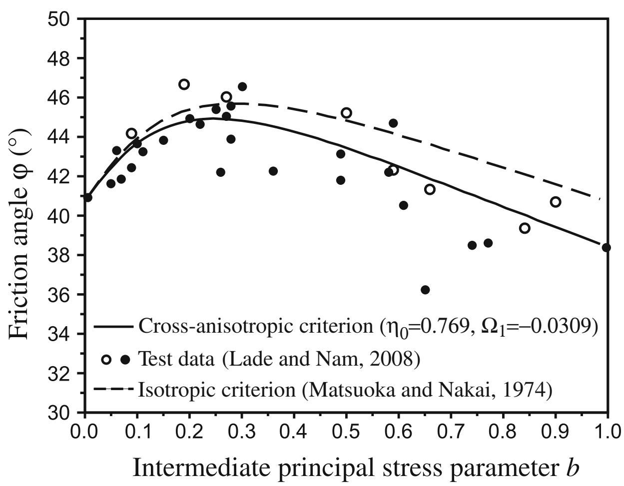
A series of triaxial tests were performed by Haruyama [5] to investigate the influence of inherent anisotropy on the deformation-strength behaviors for an assembly consisting of spherical particles of glass beads. The direction of specimen deposition coincided with that of the principal stress, σ_z. The conventional triaxial compression and conventional triaxial extension test results have been employed for the parameter determination. Shown in Fig. 8 is a comparison between the test data and the proposed cross-anisotropic failure criterion. Evidently, the proposed failure criterion can reasonably capture the influence of cross-anisotropy on the peak strength for the spherical glass beads, and generally shows a better performance than the isotropic SMP criterion.

6.1.3 *Yuubari shale [19]*

The proposed failure criterion has been applied to the prediction of shear strength on a rock, the Yuubari shale case [19]. The translated principal stresses defined in Eq. (17) have been used in the formulation. The cohesion related parameter σ_0 of rock can be derived directly from the Mohr–Coulomb cohesion and the internal friction angle [3], which is for Yuubari shale $\sigma_0 = 93$ MPa [19]. Figure 9 shows a comparison of the prediction with test data in the σ_1–σ_2 space with a constant σ_3. From this figure, it can be seen that the cross-anisotropic criterion predicted the test data better than the isotropic criterion for both $\sigma_3 = 25$ kPa, and $\sigma_3 = 50$ kPa.



(a) in normalized $\sigma_z - \sigma_\theta$-$\tau_{z\theta}$ plane



(b) in the b-φ diagram diagram

Fig. 10 Comparison of failure criteria with torsional shear test data for medium dense, cross-anisotropic Santa Monica beach sand [10] (**a**) in normalized $\sigma_z - \sigma_\theta$-$\tau_{z\theta}$ and (**b**) in the $b-\varphi$ diagram

6.1.4 Torsional shear tests

We take the case of medium dense Santa Monica beach sand tested by Lade et al. [10] as an example. Thirty-four drained torsional shear tests on this sand have been tested by Lade et al. [10]. In the torsional shear tests, the value of b is related to the inclination of σ_1 with respect to the bedding axis, β, as shown in Fig. 5, according to $b = \sin^2\beta$ [10]. Hence, $b = 0.0$, for $\beta = 0°$, and $b = 1.0$, for $\beta = 90°$. $b = 0.0$ and $b = 1.0$ correspond to triaxial compression and conventional triaxial extension respectively. Appropriate parameters have been determined in previous section for this sand. Figure 10a presents the predicted results in comparison with the test data in the normalized $\tau_{z\theta}$–$\sigma_z - \sigma_\theta$ plane. It appears that both the isotropic and the cross-anisotropic failure criteria can predicted the strength for this sand very well. However, when we compare their performance in the $\varphi-$b diagram, as shown in Fig. 10b, the improved prediction by the cross-anisotropic failure criterion is evident, as the isotropic criterion generally overestimate the friction angle in most range of b. Comparisons in Fig. 10 demonstrate that the effects of stress rotation in torsional shear tests can be well captured by the proposed failure criterion.

7 Conclusion and discussion

Based on the SMP failure criterion and the approach proposed by Pietruszczak and Mroz [17, 18], a new failure criterion for cross-anisotropic materials has been proposed in this paper. The new criterion takes full account the microstructural characteristics of the two methods, and features only two parameters which can be easily determined by either conventional triaxial tests or torsional shear tests. Application of the criterion to the prediction of shear strength for a wide range of materials including San Francisco Bay Mud [6], spherical glass beads [5], Santa Monica beach sand [10] and Yuubari shale [19] have shown the new criterion is capable of capture the strength anisotropy for these materials pretty well.

As has been mentioned earlier, Lade [8] has proposed a similar cross-anisotropic criterion based on his own isotropic criterion and the approach by Pietruszczak and Mroz [17]. Though not presenting here, our calculation shows that Eq. (21) gives a slightly smaller value of the friction angle φ than Lade's [8] criterion, except at $b = 0$. It is also emphasized that the two studies differ from each in both the expression and physical meaning of the mobilized shear stress on the left-hand side of each criterion. The current criteria stresses more the microstructural consideration which may bring more consistence with Pietruszczak and Mroz's approach. Also notably, Gao et al. [4] have developed an anisotropic failure criterion based on an isotropic failure criterion proposed by Yao et al. [22], wherein an anisotropic variable in terms of the invariants and joint invariants of the stress tensor and the fabric tensor have been introduced into the frictional coefficient of the failure criterion to account for the influence of cross-anisotropy. By introducing the microstructure tensor [17] into the SMP failure criterion, the current study evidently differs from Gao et al. [4] in both the methodology to consider the anisotropic fabric and the specific isotropic criterion based on which the anisotropic criterion has been developed. In addition, the formulation of this new criterion includes only two parameters which can be easily calibrated by conventional triaixal tests or torsional shear tests, while more than five parameters are required in the study by Gao et al. [4].

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