



## Corrigendum

## Corrigendum to “A non-coaxial critical-state model for sand accounting for fabric anisotropy and fabric evolution” [Int. J. Solids Struct, 106–107 (2017) 200–212]

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The authors regret that the Appendix of this paper contains a typo. The correct derivation of the constitutive equations is given below. The authors would like to assure that the simulation results presented in the paper have been produced using the correct formulations.

### Appendix. The constitutive equations

According to the elastic stress strain relation and equations for the plastic increment, one can get:

$$d\sigma_{ij} = E_{ijkl}d\varepsilon_{kl}^e = E_{ijkl}[d\varepsilon_{kl} - \langle L_m \rangle \kappa_{kl} - \langle L_t \rangle \kappa'_{kl}] \quad (31)$$

where  $E_{ijkl}$  is the elastic stiffness tensor expressed as

$$E_{ijkl} = \left( K - \frac{2}{3}G \right) \delta_{ij}\delta_{kl} + G(\delta_{ki}\delta_{lj} + \delta_{li}\delta_{kj}) \quad (32)$$

and

$$\kappa_{ij} = m_{ij} + \sqrt{\frac{2}{27}}D_m\delta_{ij} \quad (33)$$

$$\kappa'_{ij} = \gamma_{ij} + \sqrt{\frac{2}{27}}D_t\delta_{ij} \quad (34)$$

The condition of consistency for the yield function can be expressed as

$$df = \frac{\partial f}{\partial r_{kl}} \frac{\partial r_{kl}}{\partial \sigma_{ij}} d\sigma_{ij} + \langle L_m \rangle \frac{\partial f}{\partial H} r_h = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} - \langle L_m \rangle K_{pm} = 0 \quad (35)$$

where

$$K_{pm} = -\frac{\partial f}{\partial H} r_h = r_h. \quad (36)$$

The loading mechanism for rotation of principal stress directions can be written as

$$\underbrace{\omega n'_{kl}}_{C_{ij}} \frac{\partial r_{kl}}{\partial \sigma_{ij}} d\sigma_{ij} - \langle L_t \rangle K_{pt} = 0. \quad (37)$$

Substituting (31) into (35), one can get:

$$\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\varepsilon_{kl} - \langle L_m \rangle \underbrace{\left( \frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} \kappa_{kl} + K_{pm} \right)}_{A_{m1}} - \langle L_t \rangle \underbrace{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} \kappa'_{kl}}_{A_{t1}} = 0 \quad (38)$$

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Substituting (31) into (37), one can get:

$$C_{ij}E_{ijkl}d\varepsilon_{kl} - \langle L_m \rangle \underbrace{C_{ij}E_{ijkl}\chi_{kl}}_{A_{m2}} - \langle L_t \rangle \underbrace{(C_{ij}E_{ijkl}\chi'_{kl} + K_{pt})}_{A_{t2}} = 0 \quad (39)$$

Combing (38) and (39), the expression for  $L_m$  and  $L_t$  can be got as below:

$$L_m = \frac{C_{ij}E_{ijkl} - \frac{A_{t2}}{A_{t1}} \frac{\partial f}{\partial \sigma_{ij}} E_{ijkl}}{A_{m2} - A_{m1} \frac{A_{t2}}{A_{t1}}} d\varepsilon_{kl} = \Pi_{kl} d\varepsilon_{kl} \quad (40)$$

$$L_t = \frac{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} - C_{ij}E_{ijkl} \frac{A_{m1}}{A_{m2}}}{A_{t1} - A_{t2} \frac{A_{m1}}{A_{m2}}} d\varepsilon_{kl} = H_{kl} d\varepsilon_{kl} \quad (41)$$

Substituting Eqs. (40) and (41) into Eq. (31), one can get

$$d\sigma_{ij} = \Lambda_{ijkl} d\varepsilon_{kl} \quad (42)$$

where

$$\Lambda_{ijkl} = E_{ijkl} - h(L_m)E_{ijmn}\chi_{mn}\Pi_{kl} - h(L_t)E_{ijmn}\chi'_{mn}H_{kl} \quad (43)$$

The authors would like to apologise for any inconvenience caused.