Practical Estimation of Compression Behavior of Clayey/Silty Sands Using Equivalent Void-Ratio Concept

X. S. Shi[1] and Jidong Zhao[2]

Abstract: Clayey/silty sands are widespread as naturally sedimentary soils such as marine deposits in estuaries and offshore locations. They belong to a unique class of gap-graded soils featuring a deficiency of certain range of particle sizes and behave differently from those containing pure sand aggregates. The fines improve the stiffness of host sands, which reduces the postconstruction settlement and arching effect of foundations and dams. In this study, a simple yet effective compression model is proposed for clayey/silty sands using the equivalent void-ratio concept. A structure parameter is incorporated into the model to denote the contribution of fines on the effective force chains of gap-graded mixtures. The structure parameter is affected by the particle-size distribution and basic features of sand aggregates. It can be approximated by a constant value, which represents a combination effect of the influence factors. The limit (inactive) void ratio of clayey/silty sands decreases linearly with the increase of fine content and the structure parameter. The proposed model contains only three model parameters, all of which have clear physical meanings and can be readily calibrated based on two conventional compression tests. Simulations using the newly proposed model revealed that it is versatile to capture key features of gap-graded mixtures, including the effect of initial void ratio, interaggregate void ratio, and fine content. The performance of the proposed model is verified with tests data for six clayey sands and five silty sands (or sandy gravel). The differences between the test data and model predictions for both clayey sands and gap-graded granular mixtures are marginally small. The model can be practically useful for predicting the deformation of clayey/silty sands. **DOI:** 10.1061/(ASCE)GT.1943-5606.0002267. *© 2020 American Society of Civil Engineers.*

Author keywords: Clayey sands; Silty sands; Compression model; Interaggregate void ratio; Equivalent void-ratio concept.

Introduction

Natural soils usually contain certain proportion of fines, e.g., clay or silt (Zhao et al. 2007; Yang and Juo 2001; Simpson and Evans 2015; Guo and Cui 2020; Park and Santamarina 2017; Peng et al. 2018). The fine particles may originate from the disintegration of rock or are accumulated under flows (or gravity), and their proportion may change due to weathering and internal erosion (Chandler 2000; Cui et al. 2017; Chen et al. 2020; Zhou et al. 2017). Previous studies revealed that clayey/silty sands behave differently from those containing pure coarse sands, where the void ratio is a basic state variable controlling various behaviors of soil mixtures, e.g., the shear strength (Georgiannou et al. 1990; Salgado et al. 2000; Vallejo and Mawby 2000; Ueda et al. 2011; Ruggeri et al. 2016), compressibility (Monkul and Ozden 2007; Ham et al. 2010; Cabalar and Hasan 2013; Jiang et al. 2016; Shi and Yin 2017), and permeability (Pandian et al. 1995; Sivapullaiah et al. 2000; Watabe et al. 2011; Shi and Yin 2018). Therefore, a new model for clayey/silty sands is needed to determine their volumetric change and therefore void ratio effectively to conveniently predict the associated properties relevant to practice of geotechnical engineering.

The behavior of clayey/silty sands relies partially on fine content. At a low fine fraction, its behavior is mainly controlled by the coarse material. With an increase of fine content, the overall behavior shows a transition from coarse materials to fines, and the behavior turns to be controlled by the fines after the fine fraction exceeds a certain threshold value (Monkul and Ozden 2007; Zuo and Baudet 2015). The threshold of fine fraction distinguishing the relative dominance of coarse and fine materials is commonly named the transitional fine content (Monkul and Ozden 2007). As noted by Zuo and Baudet (2015), the transitional fine content varies between 20% and 50%, and the values determined by different methods are frequently inconsistent (Polito 1999; Dash et al. 2010; Zuo and Baudet 2015). This paper focuses on clayey/silty sands with a fine fraction below the transitional fine content. This type of soil is widespread as marine deposit in estuaries and offshore seabeds (Georgiannou et al. 1990).

The compression behavior of clayey/silty sands has been experimentally investigated extensively (Yin 1999; Ham et al. 2010; Chu et al. 2017; Shi and Yin 2017; Wu et al. 2019), with a number of models being proposed based on published data. If the fine fraction is higher than the transitional fine content, the fine particles and coarse grains can be treated as matrix and inclusions, respectively (Vallejo and Mawby 2000; Peters and Berney 2010; Zhou et al. 2016; Shi et al. 2018), and the mechanical behavior can be well modeled by mixture theory (Tandon and Weng 1988; Shi and Yin 2017; Shi et al. 2019a, b). However, as pointed out by Shi et al. (2019b), this mixture theory–based approach is not applicable to gap-graded mixtures with a fine fraction below the transitional fine content. The active and inactive void concept, originally proposed by Chang et al. (2017), provides a feasible method to describe the

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compression behavior of sand-silt mixtures. However, this model has been proposed for granular mixtures and is not applicable for clayey sands. The model proposed by Ham et al. (2010) is based on a simplified two-phase mixture theory (Gutierrez 2005). The overall void ratio of the gap-graded mixture is formulated by incorporating a model parameter representing the degree of mixing. For a given fine fraction, different initial void ratios have to be characterized by different values of a model parameter, which is not practically desirable for engineering use.

Indeed, modeling the compressibility of clayey sands has been scarcely reported in the literature. This is mainly due to the complex composition of the clayey fines. The present study makes the following original contributions for practical estimation of the compressibility of clayey/silty sands. Firstly, a unified description of the clayey sands and silty sands is formulated and clarified to provide a convenient general model for these two kinds of mixtures. Then, the equivalent void-ratio concept is extended to a finite strain range based on limited physically reasonable assumptions. Finally, a modified stiffness is adopted as a state variable for incorporating the equivalent void-ratio concept, such that the influence of fine content on the structure parameter is somehow reduced to a minimal by using the modified stiffness. Based on these considerations, a simple yet effective method has been proposed in this study for evaluating the compressibility of clayey/silty sands. It is versatile to capture the major properties of gap-graded mixtures, e.g., effects of fine fraction, initial density, and intergranular void ratio. The model parameters have clear physical meanings, and they can be easily calibrated based on a limited data of oedometer tests, which indicates a practical use of this model.

Effective Stiffness of Gap-Graded Mixtures

The clayey/silty sands are composed of sand aggregates (forming an interaggregate structure) and fines (filling the interaggregate space). The presence of fines improves the stiffness of coarse sands, which leads to some favorable engineering properties, e.g., reduce the postconstruction settlement and arching effect of foundations and dams. A practical approach is needed for evaluating the compressibility, as well as the associated engineering properties of clayey/silty sands. Because the fine content is below the transitional fine content, macropores arise, and the fines within the interaggregate space cannot hold a stable structure due to a substantial number of ratters. Classical mixture theory cannot deal with this type of structure, which calls for a new theoretical model. To this end, the equivalent void-ratio concept is adopted and extended to a finite strain case, which provides a possible description for the compressibility of clayey/silty sands.

The fines in clayey sands are usually a combination of fine sands, silts, and clay aggregates. The clay aggregate is characterized by a cluster of clay particles as adopted by Nagaraj et al. (1990), and its volume change is negligible within conventional stress range. Therefore, it behaves like solid particles, such as silts and sands, in the transmission of loading between coarse aggregates. Analogous to the granular mixtures (silty sands), the behavior of clayey sands can be also evaluated using the equivalent void-ratio concept. This provides a unified description for the mechanical behavior of both clayey sands and silty sands.

Equivalent Void-Ratio Concept

As pointed out by Cabalar and Hasan (2013), the overall void ratio is a confusing variable for describing the behavior of gap-graded mixtures because the behavior is also affected by the fine fraction. Mitchell (1976) introduced the interaggregate void ratio to unify the effect of void ratio and fine fraction. It is defined as the ratio of volume of the interaggregate voids to that of coarse solids (Monkul and Ozden 2007). For natural sedimentary soils, the coarse aggregates and fines have a similar composition because they usually originate from the same parent rock mass (Zuo and Baudet 2015; Zhou et al. 2017; Shi et al. 2019b). In this case, the interaggregate void ratio can be expressed (Thevanayagam and Mohan 2000)

$$e_g = \frac{e + f_c}{1 - f_c} \tag{1}$$

where e_g = interaggregate void ratio; e = overall void ratio of gap-graded mixtures; and f_c = fine fraction.

The interaggregate void ratio was adopted by many researchers to interpret various aspects of the mechanical behavior of gap-graded mixtures (Kuerbis et al. 1988; Georgiannou et al. 1990; Monkul and Ozden 2007; Cabalar and Hasan 2013; Deng et al. 2017). The concept of interaggregate void ratio considers that the fines are confined within the void space between coarse aggregates, i.e., fine particles are completely nonactive with no contribution to the force chains. However, a proportion of fine particles is usually wedged between coarse aggregates in the clayey/silty sands, and the fines partially participate in the form of force chains. Therefore, even a low fine content may significantly affect the behavior of clayey/silty sands (Chang and Yin 2011; Goudarzy et al. 2016; Yin et al. 2014, 2016). A notable concept allowing quantitative estimation of the mechanical properties of mixtures is the equivalent void ratio. For clayey/silty sands, the fine fraction is below the transitional fine content, and Thevanayagam et al. (2002) proposed the following equation to consider the contribution of fines on the effective force chains:

$$e^{eq} = \frac{e + (1-\lambda) f_c}{1 - (1-\lambda) f_c} \tag{2}$$

where e^{eq} = equivalent void ratio; and λ = structure parameter denoting the proportion of fines that are effective in the force chains, where λ varies between 0 and 1 with $\lambda = 0$ denoting that no fines are active in the interaggregate skeleton [Eq. (2) reduces to Eq. (1)].

Effective Stiffness of Gap-Graded Mixtures

Rahman et al. (2008) reported that the equivalent void ratio could be considered for normalizing the steady-state line of silty sands with different fine content. However, this normalization leads to some scattering from a single relationship. In this paper, the equivalent void ratio will be incorporated into the effective stiffness to capture the compression behavior of gap-graded mixtures. To this end, a reference model and the effect of fines on the modified stiffness are addressed first.

The work by Meidani et al. (2017) is employed as a reference for the compression behavior of the coarse material in gap-graded mixtures. In the study, $\mathrm{d}e/\mathrm{d}\sigma'$ changes almost linearly with the current void ratio, e, within a wide range of stress level (Meidani et al. 2017)

$$\frac{\mathrm{d}e}{\mathrm{d}\sigma'} = -\frac{\alpha}{\sigma_r} e_0 (e - e_r) \tag{3}$$

where α = model parameter; $\sigma_r = 1$ kPa and is a unit reference stress for nondimensionalizing the compression parameter α [other values (e.g., atmospheric pressure) can also be assigned to the reference stress, however, the corresponding value of α should be adjusted]; σ' = effective stress (vertical stress or effective mean stress); e_0 = initial void ratio; and e_r = inactive void ratio corresponding to the densest packing state of granular materials.

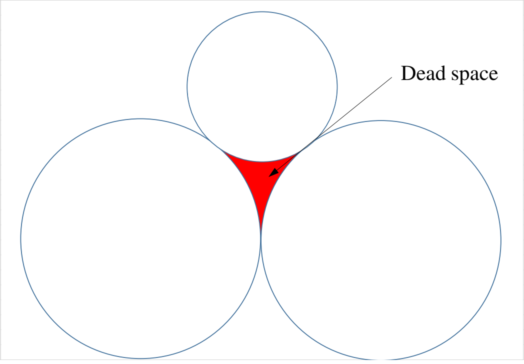

Fig. 1. Dead space around the contact between granular particles.

The model proposed by Meidani et al. (2017) is adopted as a reference due to the following reasons: (1) the model is simple but can capture the main features of the behavior of granular materials; and (2) the model parameters have clear physical meanings, and they can be easily calibrated based on a limit number of conventional oedometer tests. The model is based on the concept of active and inactive voids. The volume related to the particle rearrangement (particle sliding and rotation) is defined as active voids, whereas the inactive voids, also termed dead voids, are not reactive to the kinematic process of granular particles (Fig. 1). As mentioned by Meidani et al. (2017), the volume of dead voids relies on the particle-size distribution of host coarse materials. Therefore, the volume of inactive voids relies on particle crushing, as well as the applied stress level. The tangent stiffness of granular soils is modified in this study for the incorporation of equivalent void ratio

$$K = -\frac{\mathrm{d}\sigma'}{(1+e_0)\mathrm{d}e} = \frac{\sigma_r}{\alpha e_0(e-e_r)(1+e_0)} \tag{4}$$

The modified stiffness is proportional to the tangent stiffness of the granular materials, and the initial void ratio is incorporated for a better anticorrelation between the state variable and the initial void ratio. Other state variables correlated with the tangent stiffness can also be adopted as optional state variable. However, the value of structure may be different if other state variables are adopted. It will be seen from the validation part that the experimental data with various fine content can be well reproduced by the proposed model using a single value of structure parameter. This indicates that the influence of fine content on the structure parameter is somehow eliminated by using the modified stiffness in this study.

With reference to clayey/silty sands, the work done by previous researchers revealed a remarkable decrease in stiffness as the fine content increases (Shi et al. 2020; Rahman et al. 2012; Choo and Burns 2015; Wichtmann et al. 2015). The interaggregate void ratio e_g was adopted to interpret the effect of fines on stiffness (Choo and Burns 2015; Wichtmann et al. 2015). However, the use of interaggregate void ratio may underestimate the stiffness, especially at high fine fractions (Lashkari 2014; Yang and Liu 2016). There are several recent studies reporting that the concept of equivalent granular void ratio [Eq. (2)] can well capture the effect of fines on elastic stiffness (Rahman et al. 2014; Goudarzy et al. 2016; Yang et al. 2018). This can be done by directly replacing the void ratio in the Hardin relationship (Hardin and Black 1966) by the equivalent void ratio. The void ratio of clayey/silty sands is assumed to reproduce an equivalent modified stiffness as that of an imagined coarse-grain assembly under the same compression process. Hence, the equivalent void-ratio concept is incorporated into the modified stiffness of clayey/silty sands as follows:

$$K = -\frac{\mathrm{d}\sigma'}{(1+e_0)\mathrm{d}e} = \frac{\sigma_r}{\alpha e_0^{eq}(e^{eq}-e_r)(1+e_0^{eq})} \tag{5}$$

where e_0^{eq} = equivalent initial void ratio of gap-graded mixtures

$$e_0^{eq} = \frac{e_0+(1-\lambda)f_c}{1-(1-\lambda)f_c} \tag{6}$$

Compression Model of Gap-Graded Mixtures

Compression Model Based on Equivalent Void-Ratio Concept

From Eq. (5), one can derive the following incremental relationship between the effective stress and void ratio of gap-graded mixtures:

$$\frac{\mathrm{d}e}{\mathrm{d}\sigma'} = -\frac{\alpha e_0^{eq}(e^{eq}-e_r)(1+e_0^{eq})}{(1+e_0)\sigma_r} \tag{7}$$

Substitution of Eq. (2) into Eq. (7) gives

$$\frac{\mathrm{d}e}{\mathrm{d}\sigma'} = -\frac{\alpha e_0^{eq}(1+e_0^{eq})}{(1+e_0)\sigma_r}\left(\frac{e}{1-(1-\lambda)f_c} - \frac{e_r-(1-\lambda)f_c(1+e_r)}{1-(1-\lambda)f_c}\right) \tag{8}$$

Eq. (8) can be simplified as follows:

$$\frac{\mathrm{d}e}{\mathrm{d}\sigma'} = -\alpha^{eq}\frac{e_0^{eq}(1+e_0^{eq})}{(1+e_0)\sigma_r}(e-e_r^{eq}) \tag{9}$$

where α^{eq} = equivalent compression parameter; and e_r^{eq} = equivalent inactive void ratio of gap-graded mixtures, where α^{eq} and e_r^{eq} are functions of the fine fraction, as follows:

$$\alpha^{eq} = \frac{\alpha}{1-(1-\lambda)f_c} \tag{10a}$$

$$e_r^{eq} = e_r - (1-\lambda)f_c(1+e_r) \tag{10b}$$

Eq. (10b) indicates that the equivalent inactive void ratio of gap-graded mixtures decreases linearly with the fine fraction. This is consistent with previous work for clayey/silty sands with a fine fraction lower than the transitional fine content (Chang and Yin 2011; Chang et al. 2017).

According to Eq. (9), when the current void ratio approaches the equivalent inactive void ratio, the decrease of void ratio vanishes for further compression loading. The current void ratio of gap-graded mixtures can be derived for a given initial condition, i.e., $e = e_0$ at an infinitesimal loading. For a given initial void ratio and fine fraction, the equivalent (initial) void ratio and the equivalent inactive void ratio are constant. The incremental relationship in Eq. (9) can be integrated over the void ratio e and the effective stress σ' from the initial state to the current stress level

$$\int_{e_0}^{e}\frac{\mathrm{d}e}{(e-e_r^{eq})} = -\alpha^{eq}\frac{e_0^{eq}(1+e_0^{eq})}{(1+e_0)\sigma_r}\int_0^{\sigma'}\mathrm{d}\sigma' \tag{11}$$

Integration of Eq. (11) gives the following equation:

$$e = (e_0-e_r^{eq})\exp\left(-\alpha^{eq}\frac{e_0^{eq}(1+e_0^{eq})}{(1+e_0)\sigma_r}\sigma'\right)+e_r^{eq} \tag{12}$$

Eq. (12) is the compression model for gap-graded mixtures with a fine fraction lower than the transitional fine content,

Table 1. Parameters and void ratios for simulation of the proposed model

Series	α	e_r	λ	f_c (%)	e_0	e_g
1	0.002	0.4	0.5	0, 10, 20, 30	0.80	0.80, 1.00, 1.25, 1.57
2	0.002	0.4	0.5	0, 10, 20, 30	0.80, 0.62, 0.44, 0.26	0.80
3	0.002	0.4	0.5	20	0.40, 0.60, 0.80, 1.00	0.75, 1.00, 1.25, 1.50
4	0.002	0.4	0, 0.25, 0.50, 0.75, 1.00	20	0.80	1.25

i.e., clayey/silty sands. The void ratio depends on initial void ratio e_0 of gap-graded mixtures, inactive void ratio of coarse granular material e_r, effective stress σ', and fine fraction f_c.

Model Parameters and Their Calibration

The proposed compression model contains three parameters: α, e_r, and λ. Two of them (α and e_r) are related to the compression behavior of pure coarse material, and λ is a structure parameter of gap-graded mixtures. All have clear physical meanings: α controls the decreasing rate of the void ratio with increasing loading stress, e_r denotes the inactive void ratio and is the limit void ratio of pure coarse material, and λ represents the contribution of fines to the effective force chains of gap-graded mixtures. The structure parameter λ represents the contribution of fines on the effective force chains of interaggregate structure. It is affected by the particle-size distribution and basic features of sand aggregates (Thevanayagam and Martin 2002; Ni et al. 2004; Rahman et al. 2008; Zhao et al. 2018). In addition, λ also evolves during compression loading due to the decrease of interaggregate space. However, this effect may be not significant because the fine content is lower than the transitional fine content. The structure parameter is calibrated by a trial-and-error approach, which represents a combination effect of the mentioned factors.

A minimum of two compression tests are required for the calibration of the three parameters: one test on the pure coarse material, and the other one on a gap-graded mixture with a predefined fine fraction. Both α and e_r can be determined from the compression data of the pure coarse material. It is based on the relationship between the void ratio e and its decreasing rate $\mathrm{d}e/\mathrm{d}\sigma'$ [Eq. (3)]. For a first approximation of the compression behavior of gap-graded mixtures, it is assumed to be a constant for different fine fractions during the mechanical compression process. The structure parameter λ can be calibrated by a trial-and-error procedure based on the compression curve of a gap-graded mixture. The structure parameter calibrated based on an extremely low fine fraction may not be always reliable because the fine effect on stiffness is not distinct.

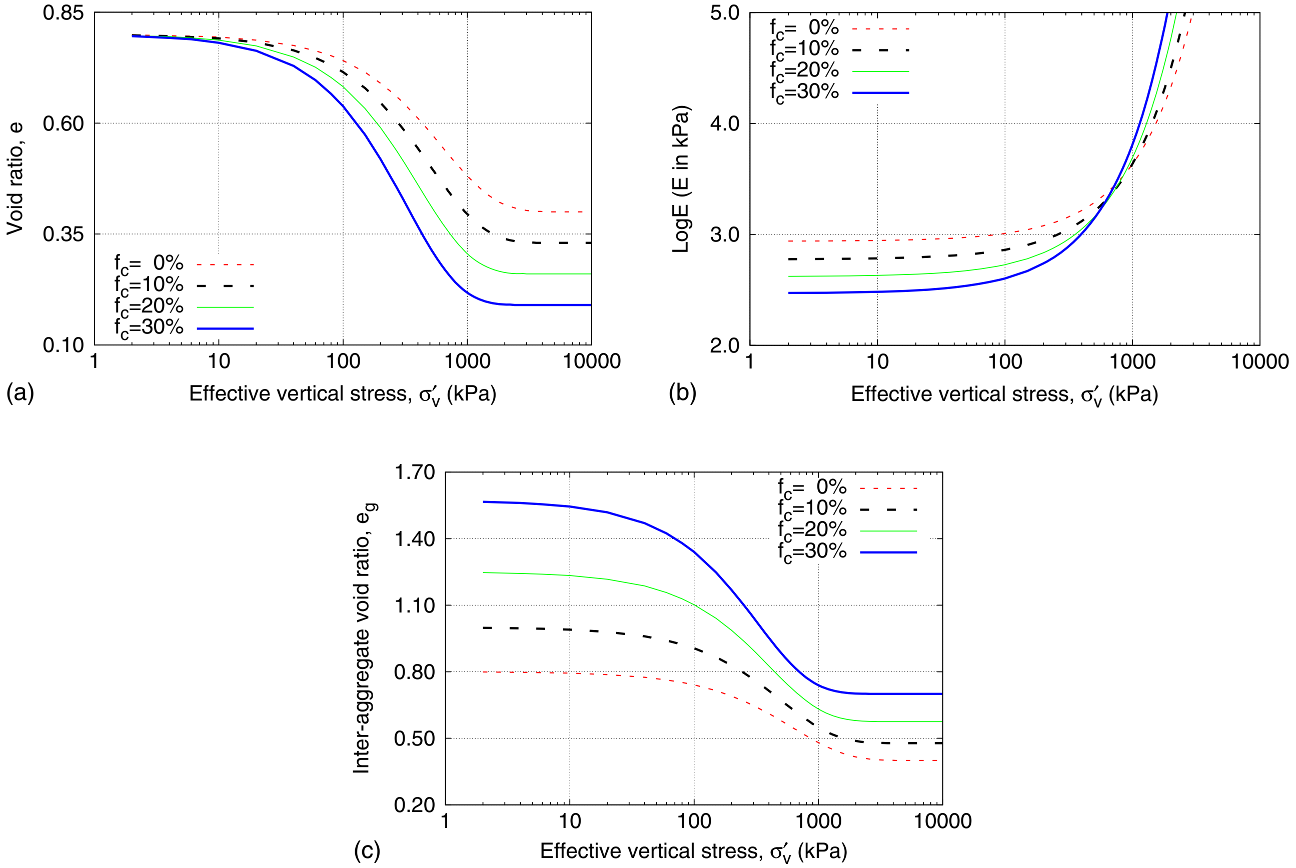

Fig. 2. Simulation of the proposed model for clayey/silty sand with different fine fractions (same initial void ratio): (a) compression curves; (b) tangent stiffness; and (c) interaggregate void ratio.

Parametric Study of the Proposed Model

From the compression model of gap-graded mixtures [Eqs. (6), (10), and (12)], the compression behavior relies on the initial void ratio and fine fraction, and the interaggregate void ratio is related to the interaggregate skeleton. In the sequel, the effects of variation of the initial void ratio (Series 1), fine fraction (Series 2), and interaggregate void ratio (Series 3) on the morphology of the proposed model is discussed. The adopted values of model parameters are listed in Table 1. The compression parameter α and inactive void ratio e_r are assumed to be 0.002 and 0.4, respectively. The structure parameter $\lambda = 0.5$ is assigned for the compression model.

The interaggregate space is filled with small particles in gap-graded soils. Therefore, the overall void ratio is usually significantly lower than that of the soils with a semiuniform particle size distribution, even below 0.25 when the fine content is close to the transitional fine content (Dash et al. 2010). The authors assume that the two phases (coarse aggregates and fines) have the same particle density. Otherwise, Eqs. (1) and (2) provide only an approximation for the state variables. A small initial stress of 2 kPa is assumed, and the sample is then loaded up to 10 MPa. The simulation results of gap-graded mixtures using the proposed model are shown in Figs. 2–4.

To investigate the effect of fine fraction, the initial void ratio is set to be 0.80 and four different fine fractions are considered (0.00, 0.10, 0.20, and 0.30). Hence, the corresponding initial values of interaggregate void ratio are 0.80, 1.00, 1.25, and 1.57, respectively. Sand-silt mixtures with the same initial void ratio but different fine fractions are seldomly reported. However, this is common for sand-clay mixtures (e.g., Mun et al. 2018). The results of mixtures with different fine contents are shown in Fig. 2, including the change of void ratio [Fig. 2(a)], tangent stiffness [Fig. 2(b)], and interaggregate void ratio [Fig. 2(c)]. It reveals that the mixtures tend to exhibit a softer response as the fine content increases, an observation consistent with Mun et al. (2018). For a given initial void ratio, adding fines increases the interaggregate void space [Fig. 2(c)] because the fines disturb the interaggregate skeleton. Therefore, the stiffness falls with an increase of fine fraction (within 1.0 MPa). However, the stiffness shows a slight increase with fine content at high stress levels. This is reasonable for gap-graded soils because the fines may partially overtake further loading due to the closure of the interaggregate void spaces.

The compression curves of gap-graded mixtures with a prescribed initial interaggregate void ratio ($e_g = 0.80$) are shown in Fig. 3. Three different fine fractions, 0.00, 0.10, and 0.20, are considered, and the corresponding initial void ratios are 0.80, 0.62, and 0.44, respectively. More fines are active in the force chains as the fine fraction increases, which overtakes additional loading. Correspondingly, for a given incremental stress, the overall tangent stiffness of the mixture increases [Fig. 3(b)], and the strain increment decreases with the rising fine fraction. Additionally, the inactive void ratio falls as fine fraction increases, which agrees with Eq. (10*b*). At a given stress level, the mixture with a higher fine fraction possesses a higher interaggregate void ratio. It seems that the fines partially overtake some further loading during the compression process.

The effect of initial void ratio (initial density) is also simulated, and the results are shown in Fig. 4. Four different initial void ratios, 0.40, 0.60, 0.80, and 1.00, are considered provided that the fine

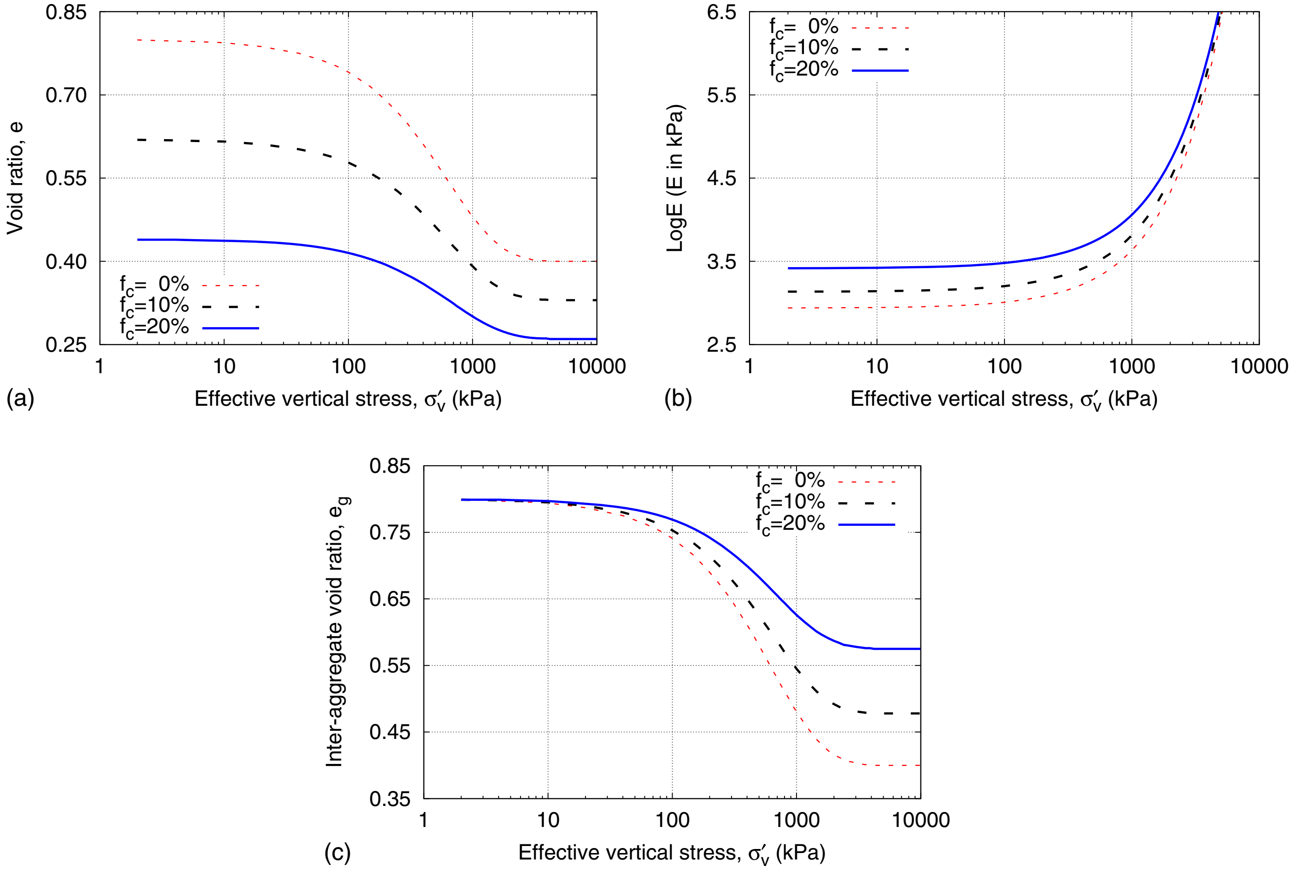

Fig. 3. Simulation of the proposed model for clayey/silty sand with different fine fractions (same initial interaggregate void ratio): (a) compression curves; (b) tangent stiffness; and (c) interaggregate void ratio.

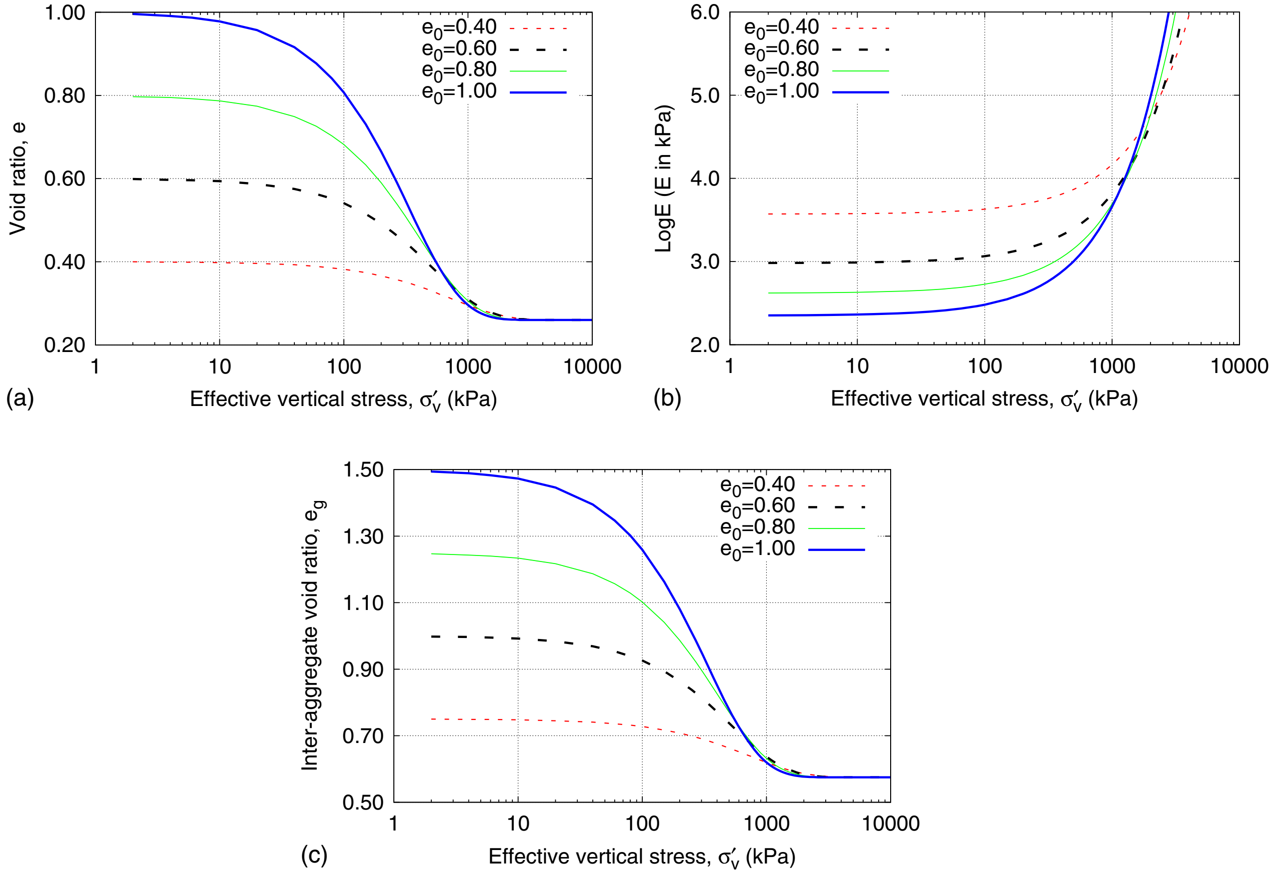

Fig. 4. Simulation of the proposed model for clayey/silty sand with different initial void ratios: (a) compression curves; (b) tangent stiffness; and (c) interaggregate void ratio.

fraction is the same (0.20). The initial interaggregate void ratios are 0.75, 1.00, 1.25, and 1.50, respectively. It is seen that the evolution of the void ratio and interaggregate void ratio is similar: because the limit (inactive) void ratios are the same, a higher initial void ratio indicates a more pronounced decreasing trend of the void ratio (or interaggregate void ratio). Consequently, the tangent stiffness decreases with the initial void ratio.

The effect of structure parameter λ on the simulated compression curves of a typical gap-graded mixture with a fine fraction of 0.20 is shown in Fig. 5. The structure parameter changes from 0 to 1.0. The assumed initial void ratios for this mixture is 0.80, and the corresponding interaggregate void ratio is 1.25. The values of model parameters are listed in Table 1 (Series 4). It can be seen from Fig. 5 that the effect of the structure parameter on the simulated compression curves is remarkable: by increasing λ, the inactive void ratio increases, which is consistent with Eq. (10*b*). It is not surprising that the proportion of fine particles wedged between coarse aggregates contributes to the change of the inactive void ratio of gap-graded soils. The calculated curves of interaggregate void move downward with an increase of the structure parameter [Fig. 5(b)], depicting a softer response of mixtures due to weaker wedging effect of fines. Therefore, the apparent yield stress (maximum curvature point of the curves) of the mixture increases with an increase of the structure parameter.

Validation of the Proposed Model

Simulation of compression curves using the proposed model in the previous section revealed that the model is versatile to capture the major properties of gap-graded mixtures, e.g., fine fraction effect and effect of initial void ratio and intergranular void ratio. Two kinds of gap-graded mixtures are used for validation of the proposed model: clayey sands and silty sands. These mixtures are abundant in the nature, including marine deposits and strongly weathered sandstones.

Clayey Sands

Compression data of six clayey sands from literature are used for validation of the proposed model. Two of them are sand mixed with natural clay (Cabalar and Hasan 2013; Mun et al. 2018), and the others are sand-kaolin mixtures (Ford 1985; Shipton and Coop 2012; Monkul and Ozden 2007). Details of the tests and physical properties of the soils are summarized as follows:

- Mason sand-Boulder clay mixtures (Mun et al. 2018): Mason sand has a specific gravity of 2.62. Its maximum and minimum void ratios are 0.78 and 0.50, respectively. Boulder clay has a specific gravity of 2.70. The liquid limit of is 41%, and the plastic limit is 18%. The soils were dried in oven before mixing, with clay contents ranging from 0% to 20%. The samples were tested at two degree of saturation, denoted as *Sr* (0.46 and 1.00), with only the saturated case adopted for validation of the model.
- Sand-Karatas clay mixtures (Cabalar and Hasan 2013): two sands, Trakya sand (TS) and crushed stone sand (CSS), are used here for validation purposes. The specific gravity of the solid particles is 2.68 for CSS and 2.65 for TS. The limit void ratios of the sands are listed in Table 2. A natural clayey soil with liquid and plastic limits of 35% and 23% are used for producing

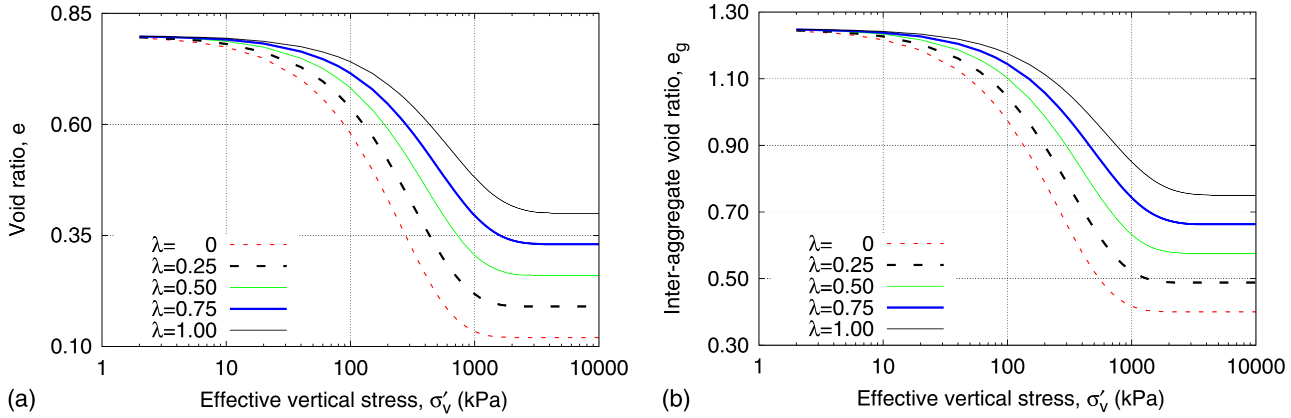

Fig. 5. Simulation of the proposed model for clayey/silty sand using different values of structure parameter: (a) compression curves; and (b) inter-aggregate void ratio.

Table 2. Details of properties of clayey sands from literature

			Limit void ratios (sand)		
Host sand	Clay	Clay fraction (%)	$e_{\max}$	$e_{\min}$	References
Mason sand	Boulder clay	0, 5, 10, 20	0.78	0.50	Mun et al. (2018)
Crushed stone sand	Karatas clay	0, 5, 10, 15, 20, 25, 30	1.10	0.57	Cabalar and Hasan (2013)
Trakya sand			0.99	0.59	
Ham River sand	Kaolin	0, 2.5, 5, 7.5, 10, 15, 20	0.90	0.45	Ford (1985)
Thames Valley sand	Kaolin	0, 10	—	—	Shipton and Coop (2012)
Sand	Kaolin	0, 10, 15, 20, 25, 30	1.12	—	Monkul and Ozden (2007)

Table 3. Properties of gap-graded granular mixtures from the literature

			Limit void ratios coarse (fines)		
Coarse materials	Fines	Fine fraction (%)	$e_{\max}$	$e_{\min}$	References
Decomposed granite	Decomposed granite	0, 30, 50	—	—	Ham et al. (2010)
Hukksund sand	Chengbei silt	0, 5, 10, 15, 20, 30	0.95 (1.41)	0.57 (0.73)	Yang et al. (2004)
Stava tailings	Stava tailings	0, 10, 30	1.07 (0.93)	0.76 (0.76)	Carrera et al. (2011)
Toyoura sand	Miled fines	0, 10, 15, 20	0.99 (1.74)	0.59 (0.61)	Zlatović and Ishihara (1995)
Leighton Buzzard sand	Mica	0, 5, 10	0.79 (3.00)	0.52 (2.17)	Cabalar (2010)

Table 4. Values of model parameters for gap-graded mixtures from the literature

	Compression parameter (host material)			
Host coarse material	α	e_r	Structure parameter (mixture), λ	References
Mason sand ($Sr = 1.00$)	1.50×10^{-4}	0.20	0.20	Mun et al. (2018)
Mason sand ($Sr = 0.46$)	9.01×10^{-5}	0.10	0.15	
Crushed stone (water)	3.00×10^{-3}	0.75	0.65	Cabalar and Hasan (2013)
Crushed stone (oil)	3.00×10^{-3}	0.75	0.71	
Trakya sand (water)	3.68×10^{-3}	0.75	0.69	
Trakya sand (oil)	2.97×10^{-3}	0.74	0.71	
Ham River sand	2.03×10^{-4}	0.74	0.00	Ford (1985)
Thames Valley sand	3.47×10^{-3}	1.45	0.10	Shipton and Coop (2012)
Sand	1.80×10^{-3}	0.70	0.33	Monkul and Ozden (2007)
Decomposed granite	1.32×10^{-4}	0.40	0.80	Ham et al. (2010)
Hukksund sand	2.30×10^{-3}	0.73	0.35	Yang et al. (2004)
Stava tailings	2.55×10^{-4}	0.57	0.75	Carrera et al. (2011)
Toyoura sand	2.71×10^{-3}	0.80	0.54	Zlatović and Ishihara (1995)
Leighton Buzzard sand	9.60×10^{-3}	0.58	0.10	Cabalar (2010)

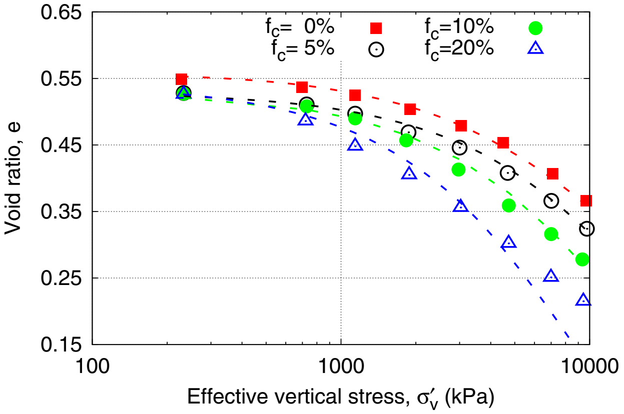

Fig. 6. Comparison of measured data and proposed model for the clayey sand from Mun et al. (2018).

the mixtures. The specific gravity of the clay particles is 2.61. Two different pore fluids, gas oil and deaired water, are used. The samples with fine fractions between 0% and 30% (Fig. 7) were prepared by using oven dried clay and sand.

- Ham River sand-Speswhite kaolin mixtures (Ford 1985, data cited from Georgiannou 1988): Ham river sand is a medium-fine sand with subangular shape. Its specific gravity is 2.66, and it has minimum void ratio of 0.450 and maximum void ratio of 0.903. The Speswhite kaolin has a specific gravity of 2.61 and Atterberg limits of 62% (liquid limit) and 32% (plastic limit). The mixtures (fine fractions below 20%) were preparing by sedimentation of the sand particles through a suspension of clay slurry.
- Thames Valley sand-Speswhite kaolin mixtures (Shipton and Coop 2012): Thames Valley sand is a poorly graded river terrace sand with similar properties as the Ham River sand (Ford 1985). The loading stress is beyond 10 MPa, much higher than those done by Ford (1985).
- Sand-kaolin mixtures (Monkul and Ozden 2007): A uniform sand with a maximum void ratio of 1.12 is used as the coarse matrix. The specific gravities of sand and clay are 2.67 and 2.61, respectively. The clay has a liquid limit of 28% and plastic limit of 19%, respectively. The mixture samples (kaolin fraction between 0% and 30%) were prepared by thoroughly mixing the oven-dried sand and clay.

Gap-Graded Granular Materials

Gap-graded granular materials are granular mixtures soils with a negligible clay fraction. Five different gap-graded mixtures from the literature are used for further validation of the proposed model, including four silty sands (Zlatović and Ishihara 1995; Yang et al. 2004; Cabalar 2010; Carrera et al. 2011) and a sandy gravel (Ham et al. 2010). Details of the coarse matrix and fines are summarized in Table 3. The maximum void ratio varies between 0.79 (mica) to 1.07 (tailings), and the minimum value are between 0.52 and 0.76. Only the samples with fine fractions below the transitional fine content are selected for verification. The transitional fine content for Leighton Buzzard sand-mica mixtures is

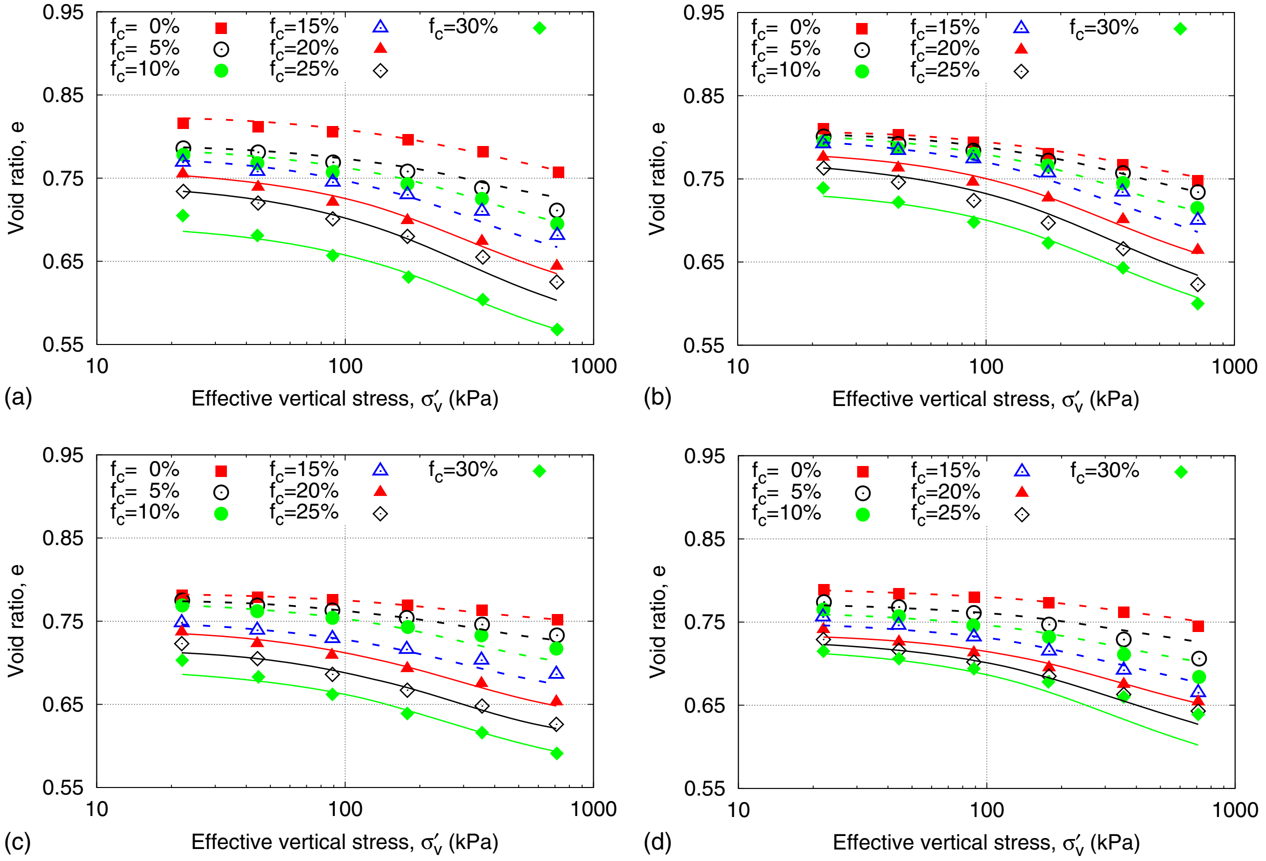

Fig. 7. Comparison of measured data and proposed model for the clayey sand from Cabalar and Hasan (2013): (a) CSS with water as pore fluid; (b) CSS with oil as pore fluid; (c) TS with water as pore fluid; and (d) TS with oil as pore fluid.

around 10%, whereas the value for decomposed granite mixtures is as high as 50%.

Model Predictions versus Experimental Data

The model parameters for the mixtures were determined from the procedure summarized in the "Compression Model of Gap-Graded Mixture" section. The values for clayey sands and gap-graded granular mixtures are given in Table 4. The compression parameter shows an extremely large variation from 9.01×10^{-5} (Mason sand) to 9.60×10^{-3} (Leighton Buzzard sand). The limit void ratio varies between 0.10 (Mason sand) to 1.45 (Thames Valley sand). The effect of pore fluids on the model parameters may be negligible. The structure parameter approximates its minimum value for some mixtures (Ham River sand-kaolin mixtures, Thames Valley sand-kaolin mixtures, and Leighton Buzzard sand-silt mixtures). This means that there is only a negligible contribution of fines to the effective force chains.

Fig. 6 presents a comparison of the measured data and this study's model predictions on clayey sand with the same initial void ratio. It is seen that the deformation of mixtures increases with fine fraction due to an increases of interaggregate void ratio after adding fines. This is consistent with the discussions in the previous section. The experimental results of other clayey sands and the predictions are presented in Figs. 7–10, revealing that the model can capture well the effect of fines and initial void ratio on the compression behavior of various clayey sands from literature, which reveals that the structure parameter is independent of the fine fraction. In this case, it can be calibrated based on the oedometer data of clayey/silty sands with an arbitrary fine content. However, the effect of fines is not so distinct for a relatively low fine content because only a limited number of fine particles contribute to the loading transmission of interaggregate structure. For a more precise prediction of the model, it is suggested to calibrate the structure parameter based on a mixture with a fine fraction higher than 20% (if data are available).

The predictions of the proposed model are compared against experimental data of three gap-graded granular mixtures as shown in Figs. 11–13 (using the values of model parameters in Table 4). It indicates that the model can well reproduce the compression behavior of gap-graded mixtures with different initial void ratios (Fig. 11) and various fine fractions. The relationship of the test data and model predictions is plotted in Fig. 14 (clayey sands) and Fig. 15 (gap-graded granular mixtures). The difference between the test data and predictions of clayey sands is less than ±0.02, and this value for gap-graded granular mixtures is ±0.03. For a better capability of the model, one can adopt different values of structure parameter for various sand fractions (Rahman et al. 2008).

Following Cabalar and Hasan (2013), different sizes and shapes of sands contribute differently to the compressibility characteristics of clayey sands. Precisely, host sands with lower roundness and

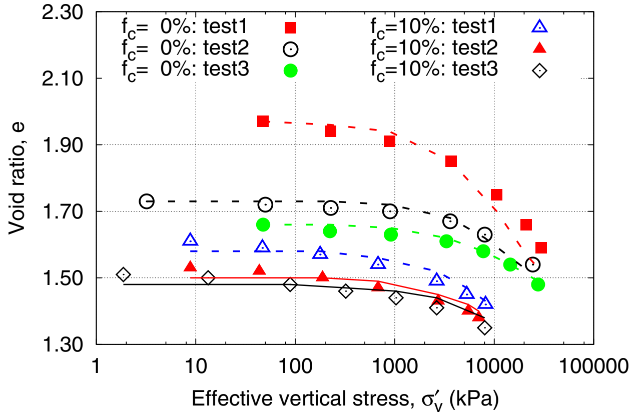

Fig. 8. Comparison of measured data and proposed model for the clayey sand from Shipton and Coop (2012).

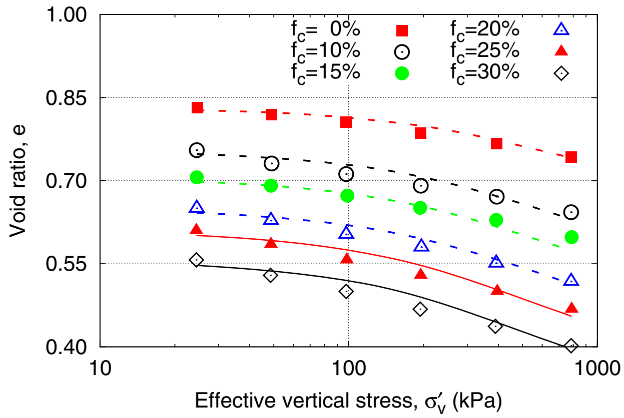

Fig. 9. Comparison of measured data and proposed model for the clayey sand from Monkul and Ozden (2007).

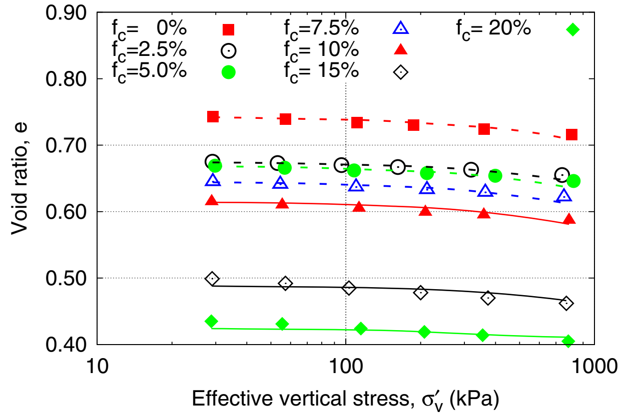

Fig. 10. Comparison of measured data and proposed model for the clayey sand from Ford (1985).

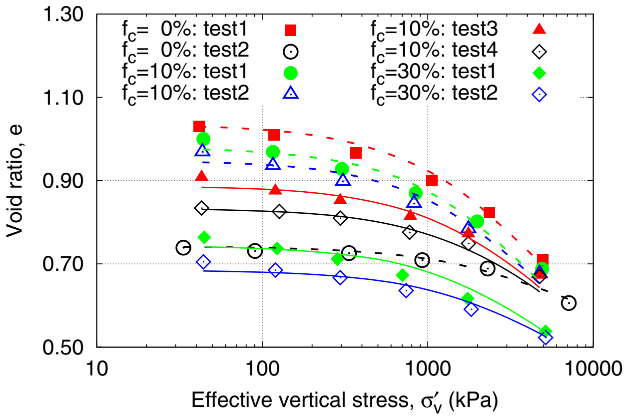

Fig. 11. Comparison of measured data and proposed model for the silty sand from Carrera et al. (2011).

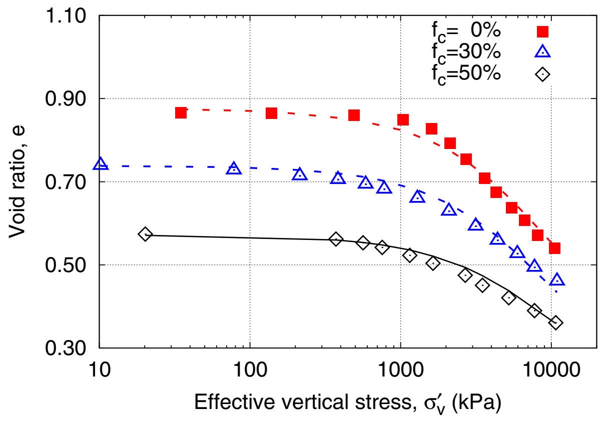

Fig. 12. Comparison of measured data and proposed model for the gap-graded granular soil from Ham et al. (2010).

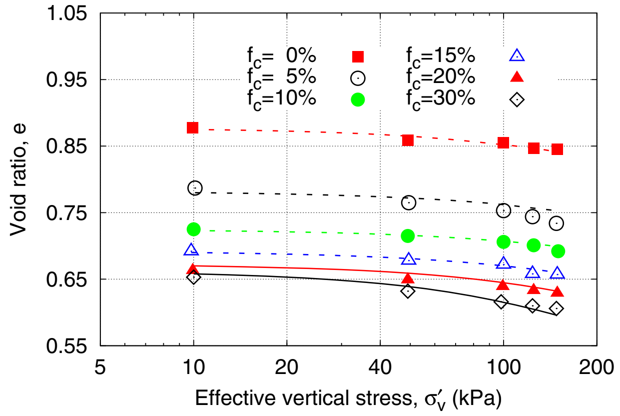

Fig. 13. Comparison of measured data and proposed model for the silty sand from Yang (2004).

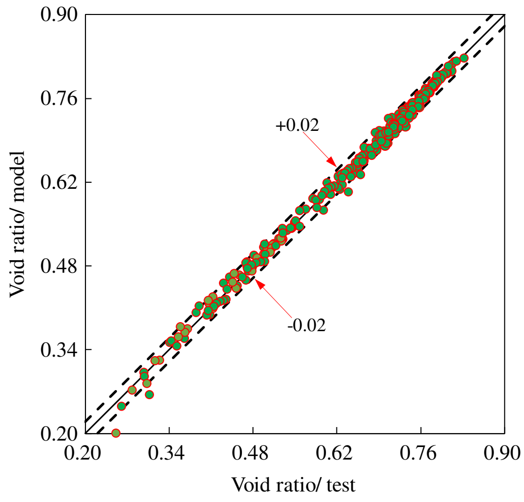

Fig. 14. Correlation between the model prediction and experimental data for clayey sands from the literature.

sphericity values usually induce a larger interaggregate space for the accommodation of the fines. As a result, the effect of fines can be negligible for a small fine fraction [e.g., 5% as mentioned by Cabalar and Hasan (2013)]. However, for host aggregates with lower roundness and sphericity values, even a small fine fraction may affect the behavior of the mixtures. In this case, the minimal fine fraction should be relatively low (smaller than 5%).

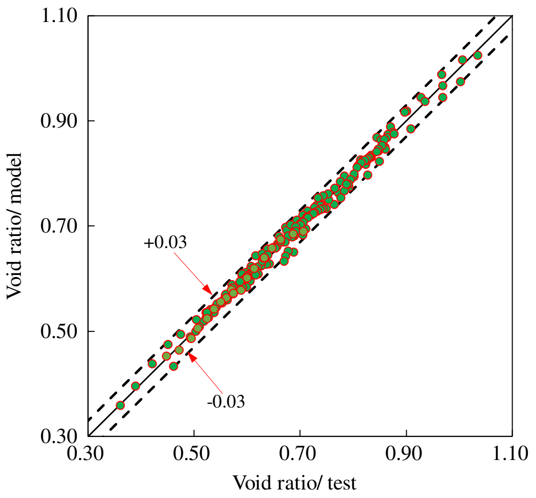

Fig. 15. Correlation between the model prediction and experimental data for gap-graded granular materials from the literature.

The proposed model has been proposed for clayey/silty sands with a fine fraction below the transitional fine content. For mixtures with a high fine fraction, they can be simplified as binary mixtures, with the fines being the matrix and the coarse material being the inclusions. In this case, the soils behaviour can be well reproduced using homogenization theory (Shi and Yin 2017; Shi et al. 2019a, b).

Conclusions

A simple yet effective compression model has been proposed for clayey/silty sands using equivalent void-ratio concept. Two physical parameters were incorporated into the model: The structure parameter denotes the contribution of fines on the effective force chains, and the inactive void ratio of clayey/silty sands decreases linearly with the increase of fine content.

The proposed model is able to capture some key features of clayey/silty sands: for a given initial void ratio, adding fines decreases the stiffness of clayey/silty sands due to the increase of interaggregate void spaces. However, if the interaggregate void ratio is prescribed, the stiffness increases with the fine content because the fines contribute to the interaggregate structure.

The proposed model contains only three model parameters, which are readily able to be calibrated based on two compression tests. The performance of the proposed model was verified by comparing the model predictions with tests data for six types of clayey sands and five types of silty sands (or sandy gravel). The comparison showed a good agreement, proving a feasible future for the proposed model to effectively predict the void ratio of clayey/silty sands with only a limited number of model parameters.

Data Availability Statement

Some or all data, models, or code generated or used during the study are available in a repository or online in accordance with funder data retention policies:

1. Compression data of clayey sand (Mun et al. 2018). http://gcf-conf.ust.hk/unsat2018/paper/pdf/1a-19.UNSAT2018_389.pdf

2. Compression data of clayey sand (Cabalar and Hasan 2013). https://www.sciencedirect.com/science/article/pii/S0013795213002068
3. Compression data of clayey sand (Ford 1985, cited from Georgiannou 1988). https://spiral.imperial.ac.uk/bitstream/10044/1/7367/1/Vasiliki_Nikolaou_Georgiannou-1989-PhD-Thesis.pdf
4. Compression data of clayey sand (Shipton and Coop 2012). https://www.sciencedirect.com/science/article/pii/S0038080612000789
5. Compression data of clayey sand (Monkul and Ozden 2007). https://www.sciencedirect.com/science/article/pii/S0013795206002833
6. Compression data of decomposed granite (Ham et al. 2010). https://ascelibrary.org/doi/full/10.1061/%28ASCE%29GT.1943-5606.0000370
7. Compression data of silty sands (Yang. 2004, cited from Chang et al. 2017). https://link.springer.com/article/10.1007/s11440-017-0598-1
8. Compression data of silty sands (Carrera et al. 2011). https://www.icevirtuallibrary.com/doi/full/10.1680/geot.9.P.009
9. Compression data of silty sands (Carrera et al. 2011). https://www.researchgate.net/profile/Mehrashk_Meidani/post/Does_anyone_know_the_void_ratio_of_Toyoura_sand_with_fines/attachment/59d646b8c49f478072eae95e/AS:273836770037775@1442299181733/download/%5B1995+Zlatovic+and+Ishihara%5D+on+the+influence+of+nonplastic+fines+on+residual+strength.pdf
10. Compression data of silty sands (Cabalar et al. 2010). https://www.sciencedirect.com/science/article/pii/S0013795210000050

Some or all data, models, or code generated or used during the study are available from the corresponding author by request: Simulations of the proposed model, data in Figs. 2–5.

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